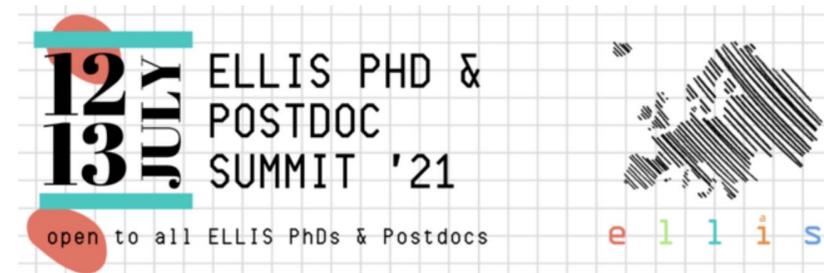
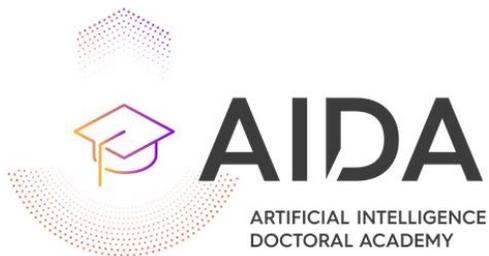
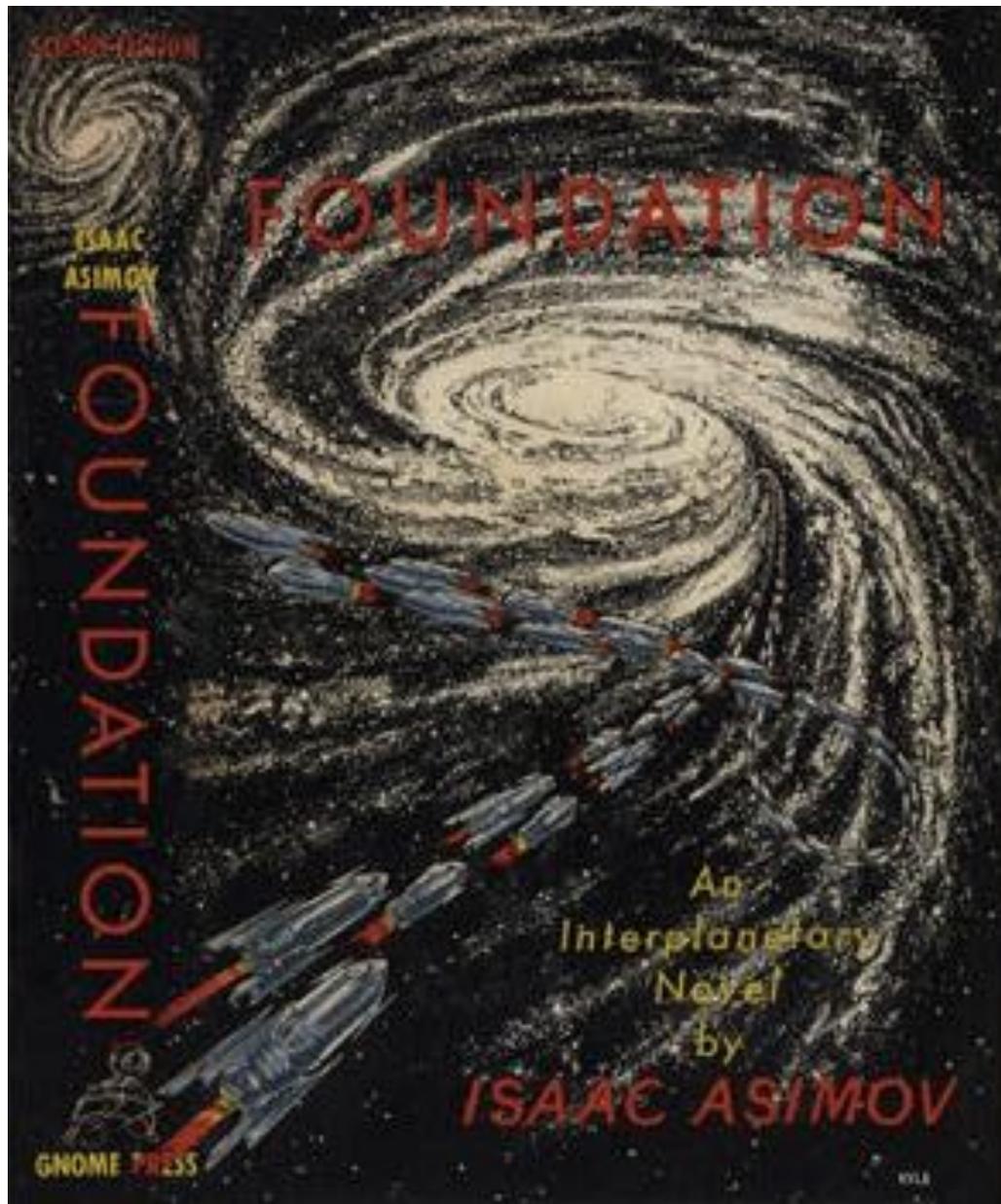


# Symbolic, Statistical, and Causal Representations

Bernhard Schölkopf

Max Planck Institute for Intelligent Systems





*“Hari Seldon [..] brought the science of psychohistory to its full development. [..] The individual human being is unpredictable, but the reactions of human mobs, Seldon found, could be treated statistically.”*





## NEW NAVY DEVICE LEARNS BY DOING

Psychologist Shows Embryo  
of Computer Designed to  
Read and Grow Wiser

Dr. Frank Rosenblatt, designer of the Perceptron, conducted the demonstration. He said the machine would be the first device to think as the human brain. As do human beings, Perceptron will make mistakes at first, but will grow wiser as it gains experience, he

Later Perceptrons will be able to recognize people and call out their names and instantly translate speech in one language to speech or writing in another language, it was predicted.

THE NEW

## Books o

By (

"If this were an entirely accurate count of my life in Cork," the of "Mrs. O."\* tells us, "I should ably be writing it behind bars. So I say that it is impressionistically true not always factually so."

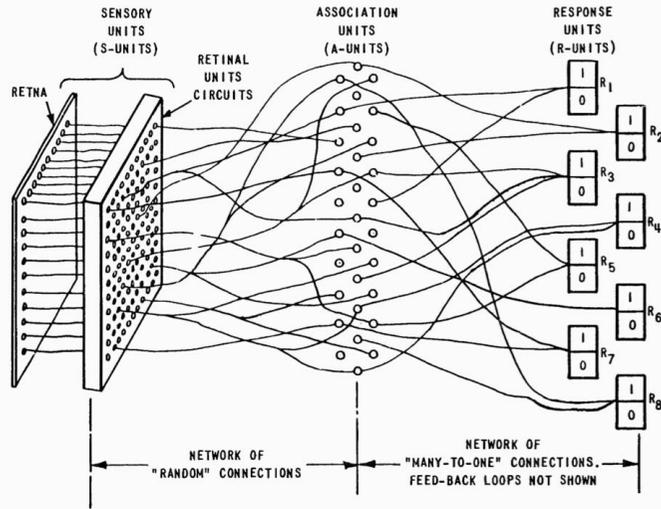


Figure 1 ORGANIZATION OF THE MARK I PERCEPTRON

# perceptron



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buy it  
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us to his

The problems of running a pub in Cork were often hilarious, seldom businesslike and sometimes tragic. The gamut of life she saw was as various as the life you will encounter on Manhattan Island if you follow Park

what is fed into them on punch cards or magnetic tape.

Later Perceptrons will be able to recognize people and call out their names and instantly translate speech in one language to

name stamped on it in gold.  
Trade Mark

PERSONAL  
SAVINGS LOANS



# Symbolic AI

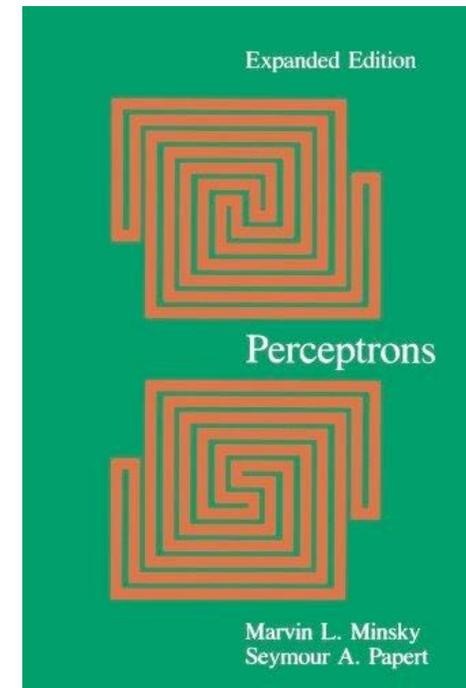
Newell & Simon:

*The Physical Symbol System Hypothesis.* A physical symbol system has the necessary and sufficient means for general intelligent action.

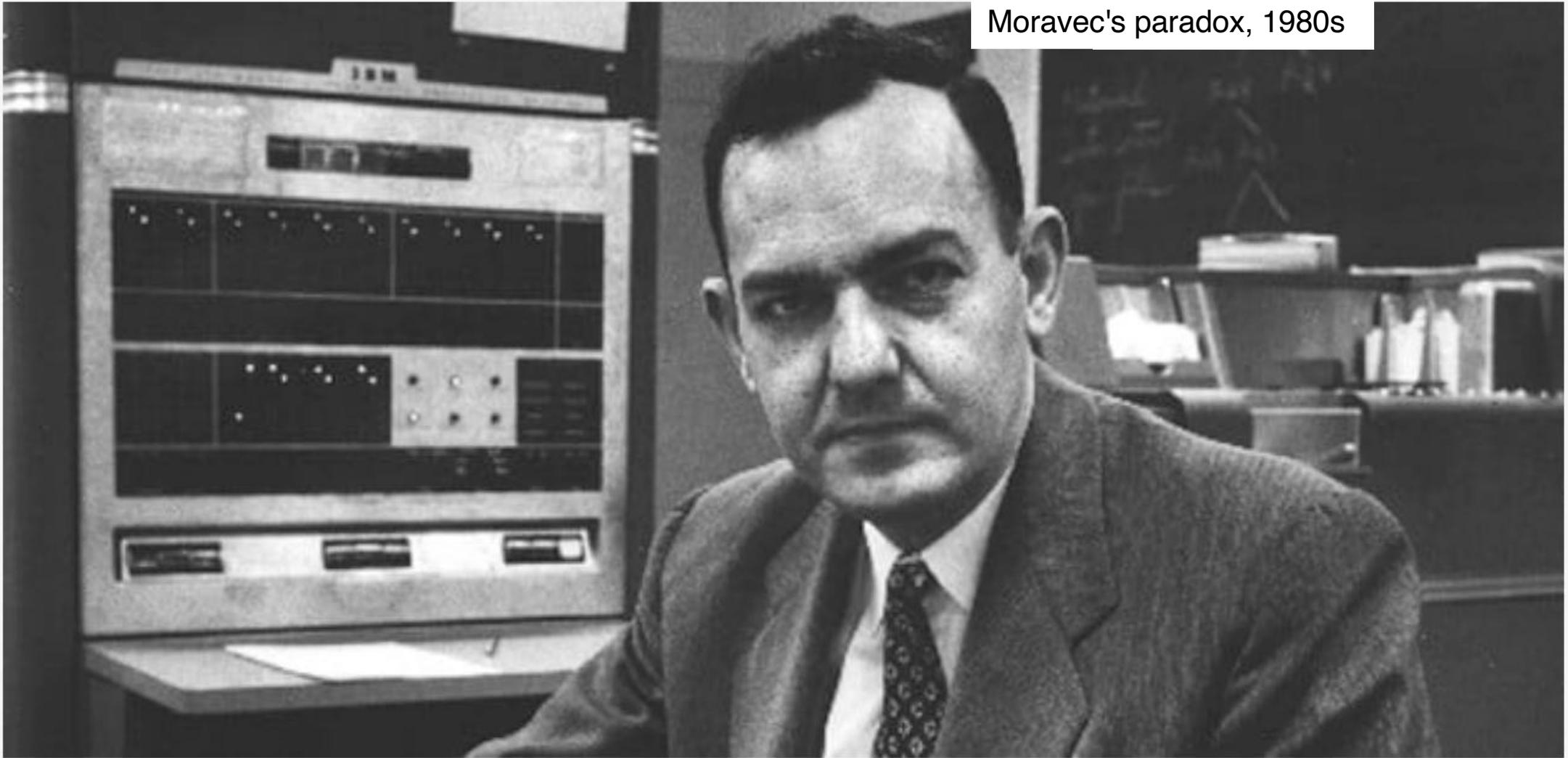


1975  
Turing  
Award  
Lecture

Bernhard Schölkopf



Moravec's paradox, 1980s



**“Machines will be capable, within twenty years, of doing any work a man can do”**

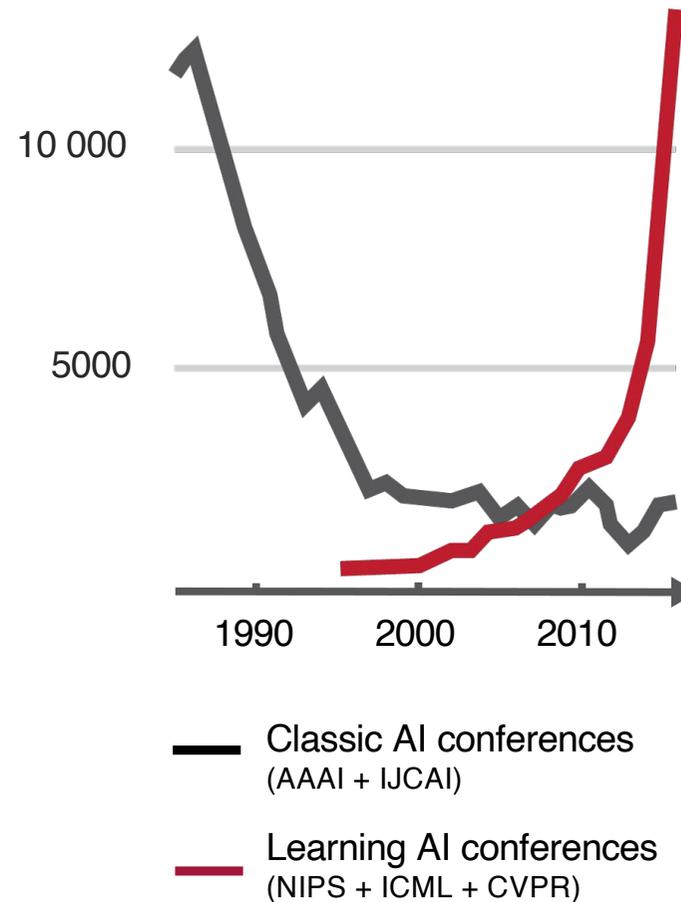
## Classic AI



Programmed by humans

Bernhard Schölkopf

## Learning AI (Machine learning)



Programmed by humans and learning from experience

(Video: D. Büchler)

Image credit: W. Brendel / M. Bethge





# THE NEURAL NET TANK URBAN LEGEND

AI folklore tells a story about a neural network trained to detect tanks which instead learned to detect time of day; investigating, this probably never happened.

[NN](#), [history](#), [sociology](#), [Google](#), [bibliography](#)

20 Sep 2011–14 Aug 2019 · finished · [certainty: highly likely](#) · [importance: 4](#)

SITE

ME

NEW:

◦ MAIL

◦ /R/GWERN

SUPPORT ON PATREON

## 1 Did It Happen?

### 1.1 Versions of the Story

1.1.1 2010s

1.1.2 2000s

1.1.3 1990s

1.1.4 1980s

1.1.5 1960s

### 1.2 Evaluation

1.2.1 Sourcing

1.2.2 Variations

1.2.3 Urban Legends

1.2.4 Origin

## 2 Could it Happen?

### 2.1 Could Something Like it Happen?

## 3 Should We Tell Stories We Know Aren't True?

### 3.1 Alternative examples

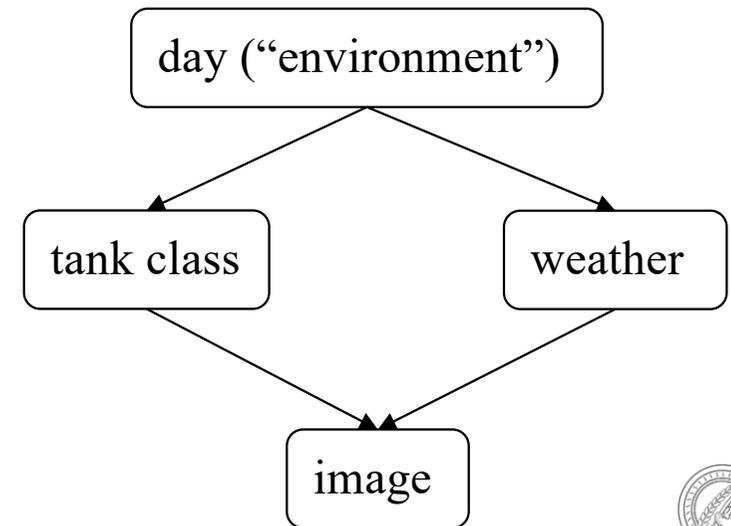
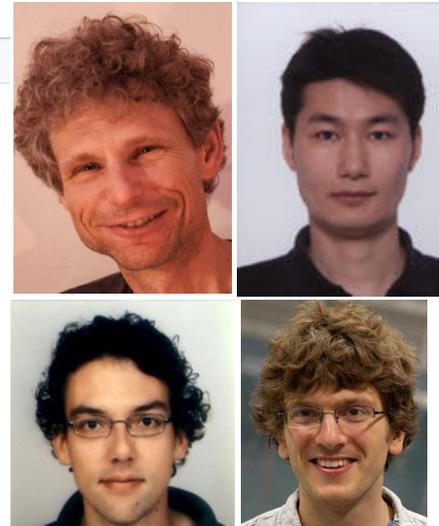
## 4 See Also

## 5 External Links

A cautionary tale in artificial intelligence tells about researchers training an neural network (NN) to detect tanks in photographs, succeeding, only to realize the photographs had been collected under specific conditions for tanks/non-tanks and the NN had learned something useless like time of day. This story is often told to warn about the limits of algorithms and importance of data collection to avoid “dataset bias”/“data leakage” where the collected data can be solved using algorithms that do not generalize to the true data distribution, but the tank story is usually never sourced.

I collate many extent versions dating back a quarter of a century to 1992 along with two NN-related anecdotes from the 1960s; their contradictions & details indicate a classic “urban legend”, with a probable origin in a speculative question in the 1960s by Edward Fredkin at an AI conference about some early NN research, which was subsequently classified & never followed up on.

I suggest that dataset bias is real but exaggerated by the tank story, giving a misleading indication of risks from deep learning and that it would be better to not repeat it but use real examples of dataset bias and focus on larger-scale risks like AI systems optimizing for wrong utility functions.



# Human-level object recognition?



cow milk agriculture farm cattle livestock dairy  
beef hayfield field grass mammal pasture calf  
farmland rural animal pastoral bull grassland



cow beef agriculture cattle milk pasture mammal  
livestock farmland grass farm hayfield rural herd  
dairy pastoral grassland field calf bull



cow mammal pasture grass animal no person nature  
agriculture livestock hayfield cattle farm rural field  
milk grassland beef pastoral countryside

*from Perona, 2017;  
cf. Lopez-Paz et al., 2016*

# Machine learning uses correlations rather than causality



beach sand travel no person water sea seashore  
summer sky outdoors ocean nature



no person water mammal cattle outdoors cow  
landscape travel sky livestock



water no person beach seashore sea sand mammal  
outdoors travel ocean surf sky

*from Perona, 2017;  
cf. Lopez-Paz et al., 2016*

# Adversarial Vulnerability

“pig”



+ 0.005 x



=

“airliner”



Image credit: [http://people.csail.mit.edu/madry/lab/blog/adversarial/2018/07/06/adversarial\\_intro/](http://people.csail.mit.edu/madry/lab/blog/adversarial/2018/07/06/adversarial_intro/)

C. Szegedy et al. Intriguing properties of neural networks. *arXiv:1312.6199*, 2013

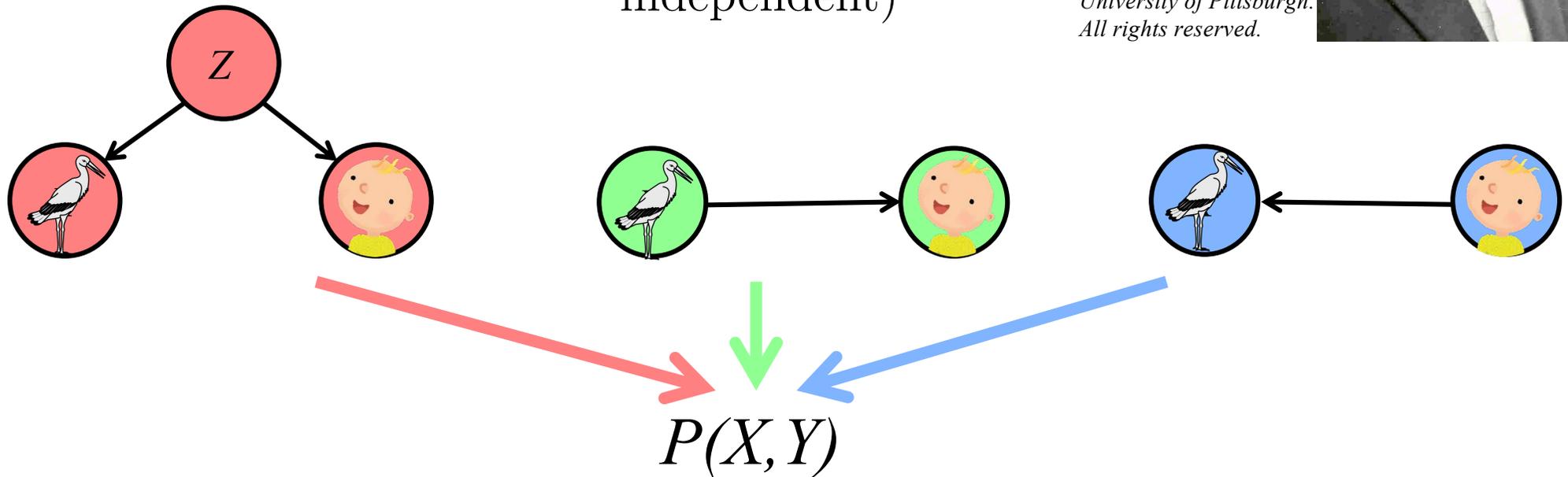
# Reichenbach's Common Cause Principle

(i) if  $X$  and  $Y$  are dependent, then there exists  $Z$  *causally* influencing both;

(ii)  $Z$  screens  $X$  and  $Y$  from each other (given  $Z$ ,  $X$  and  $Y$  become independent)



by permission of the University of Pittsburgh. All rights reserved.



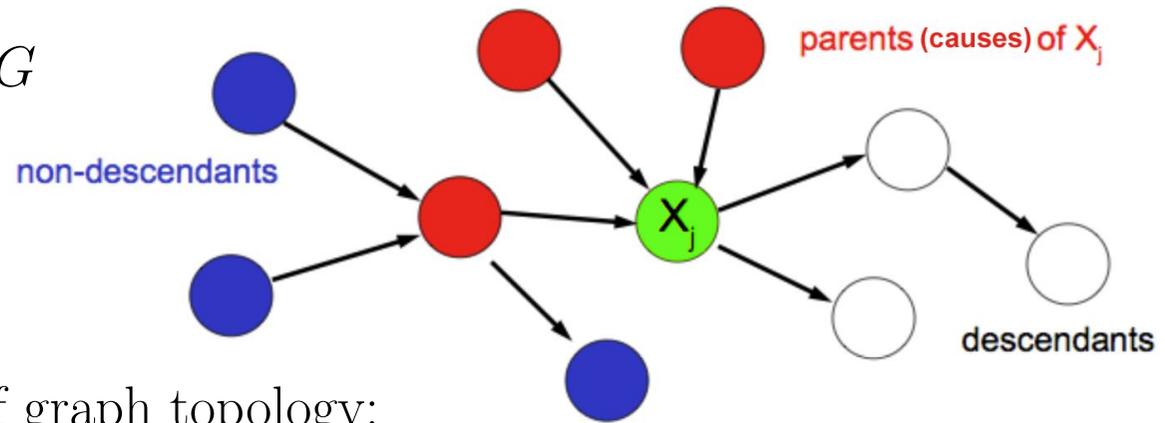
$$\sum_z p(x|z)p(y|z)p(z)$$

$$p(x)p(y|x)$$

$$p(x|y)p(y)$$

# Structural causal models (Pearl, Spirtes, et al.)

- Set of observables  $X_1, \dots, X_n$  on a DAG  $G$
- arrows represent direct causation
- $X_i := f_i(\text{PA}_i, U_i)$  with independent RVs  $U_1, \dots, U_n$ .
- distribution of the  $U_i$  picks up footprint of graph topology:  
*observational distribution*  $p(X_1, \dots, X_n)$  satisfies:



Conditioned on its **parents**,  $X_i$  is independent of its **non-descendants** (*causal Markov condition*)

—assay using conditional independence testing (for  $n > 2$ )

- $(G, p)$  is a “graphical model” (Lauritzen, 1996),  $p(X_1, \dots, X_n) = \prod_i p(X_i | \text{Parents}_i)$
- *interventions / mechanism shifts* are modelled by changing functions (**mechanisms**); entail *interventional distribution*

# Causality in differential equations

Consider the set of differential equations

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^d,$$

with initial value  $\mathbf{x}(t_0) = \mathbf{x}_0$ .

**Picard–Lindelöf:** locally, if  $f$  is Lipschitz, there exists a unique solution  $\mathbf{x}(t)$

$\implies$  the immediate future of  $\mathbf{x}$  is implied by its past

Using  $dt$  and  $d\mathbf{x} = \mathbf{x}(t + dt) - \mathbf{x}(t)$ :

$$\mathbf{x}(t + dt) = \mathbf{x}(t) + dt \cdot f(\mathbf{x}(t)).$$

This tells us which entries of  $\mathbf{x}(t)$  cause the future of others  $\mathbf{x}(t + dt)$ , i.e., the causal structure.

# Linking ODEs and SCMs

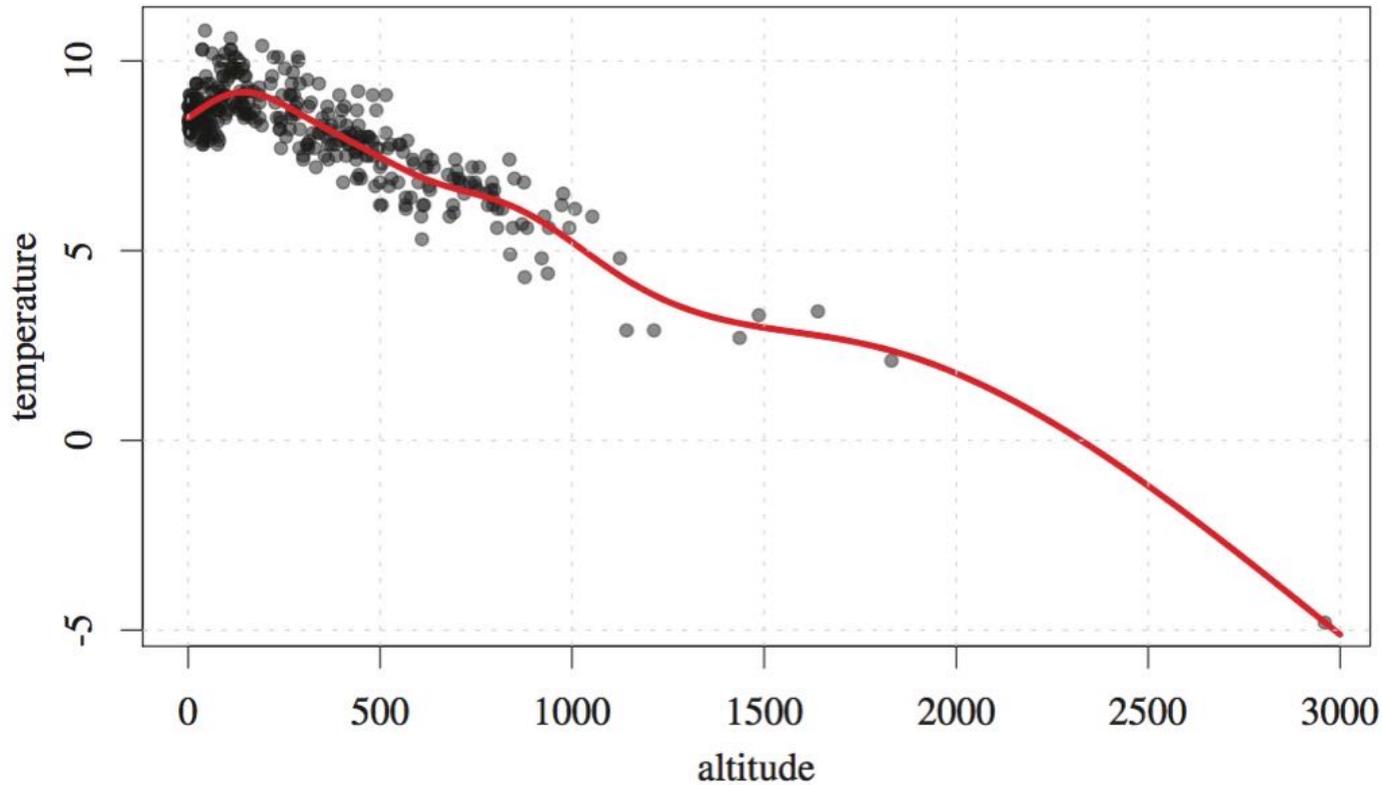
There is no reason why simple SCMs should be derivable in general.

Derive SCMs describing the interventional behavior of a coupled ODE system in equilibrium state and perturbed in an “adiabatic” way (Mooij et al., UAI 2013)

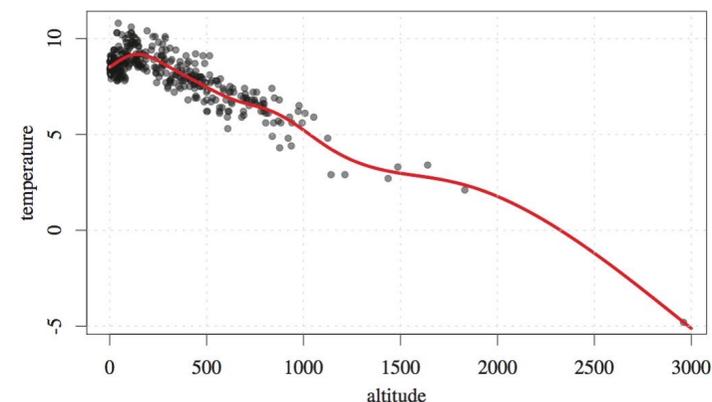
Generalization to oscillatory systems (Rubenstein et al., UAI 2018)

Subject to appropriate conditions, structural models can arise from coarse-graining of microscopic models, including microscopic structural equation models (Chalupka et al. 2015; Rubenstein et al., UAI 2017) or temporally aggregated time series (Gong et al., UAI 2017).

# What is cause and what is effect?



$$\begin{aligned} p(a,t) &= p(a|t) p(t) & T \rightarrow A \\ &= p(t|a) p(a) & A \rightarrow T \end{aligned}$$



- **intervention** on  $a$ : raise the city, find that  $t$  changes
- hypothetical intervention on  $a$ : still expect that  $t$  changes, since we can think of a physical mechanism  $p(t|a)$  that is **independent** of  $p(a)$
- we expect that  $p(t|a)$  is **invariant** across, say, different countries in a similar climate zone

## *Independent Causal Mechanisms*

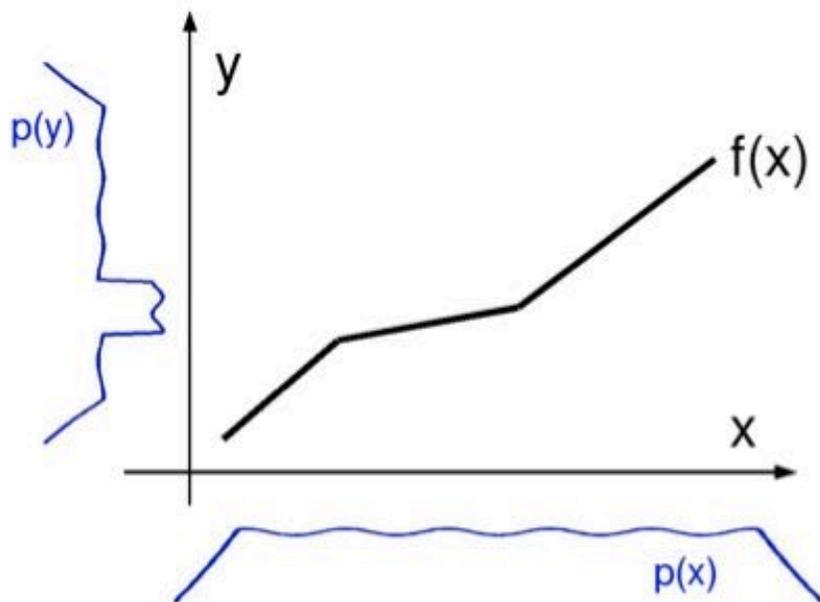
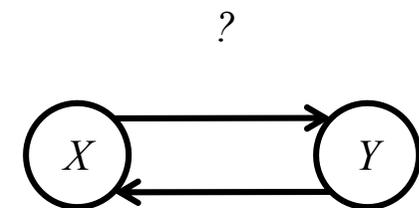
### *Principle (ICM):*

*The causal generative process is composed of autonomous modules that do not inform or influence each other.*



# Independence of input and mechanism

- No noise on effect variable
- Assumption:  $y = f(x)$  with invertible  $f$



*Daniusis, Janzing, Mooij, Zscheischler, Steudel, Zhang, Schölkopf:*

Inferring deterministic causal relations, *UAI* 2010



# Causal independence implies anticausal dependence

Assume that  $f$  is a monotonically increasing bijection of  $[0, 1]$ .

View  $p_x$  and  $\log f'$  as RVs on the prob. space  $[0, 1]$  w. Lebesgue measure.

**Postulate (independence of mechanism and input):**

$$\text{Cov}(\log f', p_x) = 0$$

**Note:** this is equivalent to

$$\int_0^1 \log f'(x) p(x) dx = \int_0^1 \log f'(x) dx,$$

since  $\text{Cov}(\log f', p_x) = E[\log f' \cdot p_x] - E[\log f'] E[p_x] = E[\log f' \cdot p_x] - E[\log f']$ .

**Proposition:** If  $f \neq Id$ ,

$$\text{Cov}(\log f^{-1'}, p_y) > 0.$$

$u_x, u_y$  uniform densities for  $x, y$

$v_x, v_y$  densities for  $x, y$  induced by transforming  $u_y, u_x$  via  $f^{-1}$  and  $f$

Equivalent formulations of the postulate:

*Additivity of Entropy:*

$$S(p_y) - S(p_x) = S(v_y) - S(u_x)$$

*Orthogonality (information geometric):*

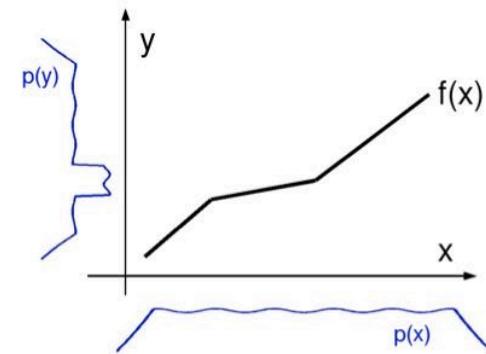
$$D(p_x \parallel v_x) = D(p_x \parallel u_x) + D(u_x \parallel v_x)$$

which can be rewritten as

$$D(p_y \parallel u_y) = D(p_x \parallel u_x) + D(v_y \parallel u_y)$$

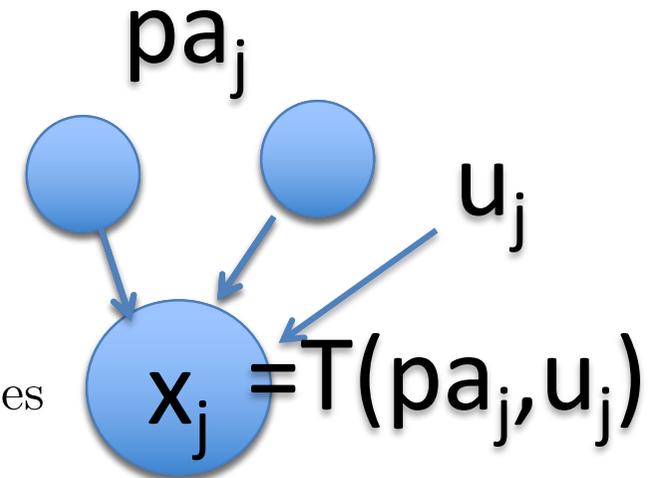
*Interpretation:*

irregularity of  $p_y$  = irregularity of  $p_x$  + irregularity introduced by  $f$



# Algorithmic structural causal model

- for every node  $x_j$  there exists a program  $u_j$  that computes  $x_j$  from its parents  $pa_j$
- all  $u_j$  are jointly independent
- the program  $u_j$  represents the causal mechanism that generates the effect from its causes
- $u_j$  are the analog of the unobserved noise terms in the statistical functional model

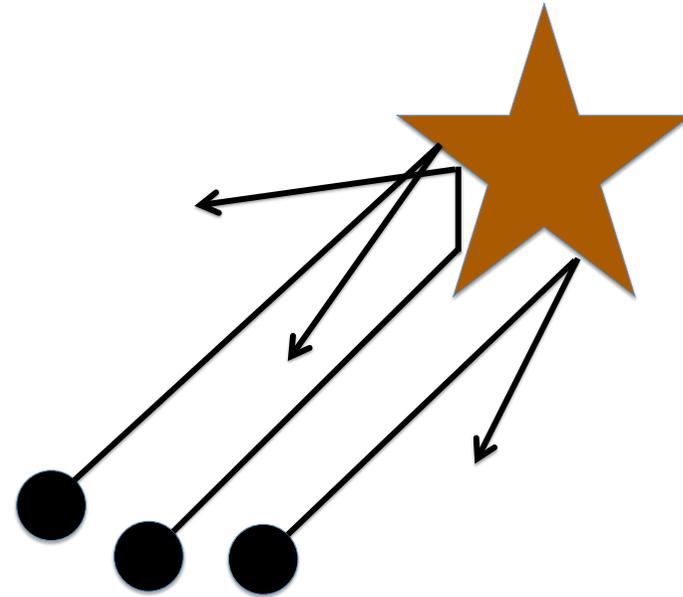


**Theorem:** this model implies the causal Markov condition (replacing Shannon entropy with Kolmogorov complexity).

*(Janzing & Schölkopf, IEEE Trans. Information Theory 2010)*

# Gedankenexperiment

Particles scattered at an object



- incoming beam: ‘cause’
- scattering at object: ‘mechanism’
- outgoing beam: ‘effect’, contains information about the object

# Independence assumption

- $s$  initial state of a physical system
- $M$  the system dynamics applied for some fixed time

**Independence Principle:**  $s$  and  $M$  are algorithmically independent

$$I(s : M) \stackrel{\pm}{=} 0,$$

i.e., knowing  $s$  does not enable a shorter description of  $M$  and vice versa.

# Thermodynamic Arrow of Time

**Theorem [non-decrease of entropy].** Let  $M$  be a bijective map on the set of states of a system then  $I(s : M) \stackrel{+}{=} 0$  implies

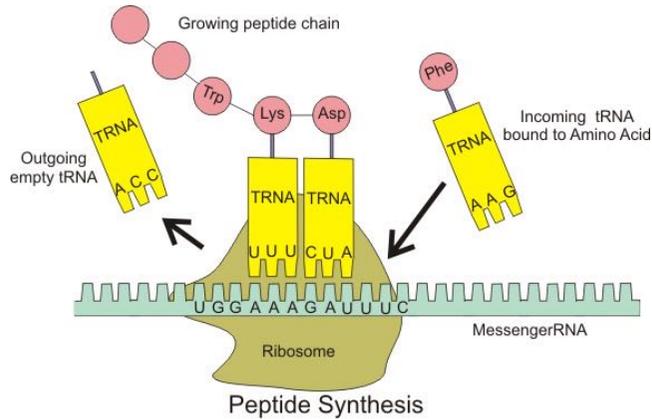
$$K(M(s)) \stackrel{+}{\geq} K(s)$$

Proof idea: If  $M(s)$  admits a shorter description than  $s$ , knowing  $M$  admits a shorter description of  $s$ : just describe  $M(s)$  and then apply  $M^{-1}$ .

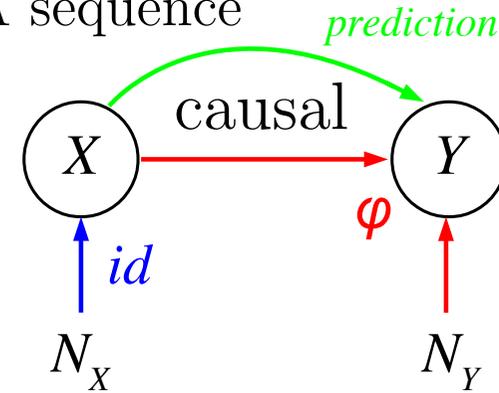
*Janzing, Chaves, Schölkopf.* Algorithmic independence of initial condition and dynamical law in thermodynamics and causal inference. New J. of Physics, 2016

# Using cause-effect knowledge

- example 1: predict protein from mRNA sequence

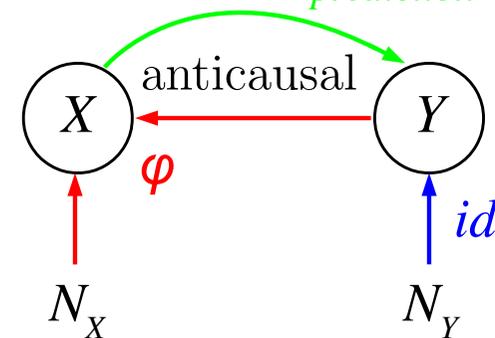
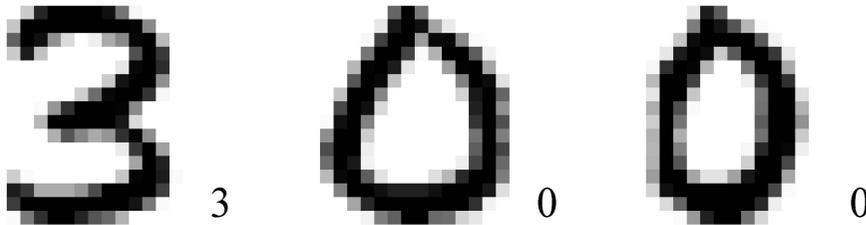


Source: [http://commons.wikimedia.org/wiki/File:Peptide\\_syn.png](http://commons.wikimedia.org/wiki/File:Peptide_syn.png)



causal mechanism  $\varphi$

- example 2: predict class membership from handwritten digit



# Covariate Shift and Semi-Supervised Learning

**Assumption:**  $p(C)$  and mechanism  $p(E|C)$  “independent”

**Goal:** learn  $X \mapsto Y$ , i.e., estimate (properties of)  $p(Y|X)$

*Semi-supervised learning:* improve estimate by more data from  $p(X)$

*Covariate shift:*  $p(X)$  changes between training and test

## Causal learning

$p(X)$  and  $p(Y|X)$  independent

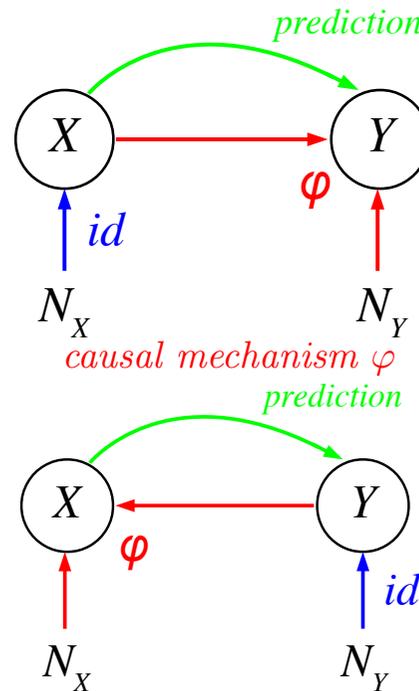
1. semi-supervised learning impossible
2.  $p(Y|X)$  invariant under change in  $p(X)$

## Anticausal learning

$p(Y)$  and  $p(X|Y)$  independent

hence  $p(X)$  and  $p(Y|X)$  dependent

1. semi-supervised learning possible
2.  $p(Y|X)$  changes with  $p(X)$



Schölkopf, Janzing, Peters, Sgouritsa, Zhang, Mooij, 2012, cf. Storkey, 2009; Bareinboim & Pearl, 2012

- Experimental Meta-Analysis confirms prediction

*Schölkopf et al., ICML 2012; von Kügelgen et al., UAI 2020, Jin et al., submitted*

- All known SSL assumptions link  $p(X)$  to  $p(Y|X)$ :

- **Cluster assumption:** points in same cluster of  $p(X)$  have the same  $Y$
- **Low density separation assumption:**  $p(Y|X)$  should cross 0.5 in an area where  $p(X)$  is small
- **Semi-supervised smoothness assumption:**  $E(Y|X)$  should be smooth where  $p(X)$  is large

# Independent Causal Mechanisms in NLP

(with Zhijing Jin & Julius von Kügelgen)



Prompt for annotators

? Given the English sentence above, can you write its Spanish translation?

Cause: [En] This is a beautiful world.

Effect: [Es] Este es un mundo hermoso.

Annotation process  
(Noise)

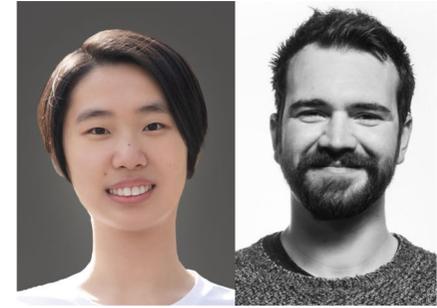
Effect = CausalMechanism (Cause, Noise)

## Common NLP tasks:

Category	Example NLP Tasks
Causal learning	Summarization, question answering, parsing, tagging, data-to-text generation, information extraction
Anticausal learning	Author attribute classification, question generation, review sentiment classification
Other/mixed (depending on data collection)	Machine translation, language modeling, intent classification

# ICM in NLP: Findings

(with Zhijing Jin & Julius von Kügelgen)

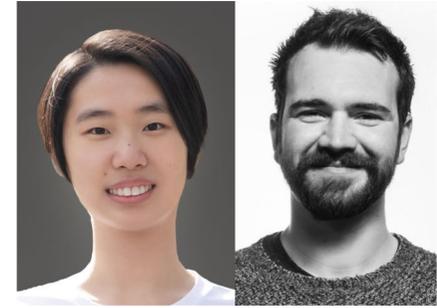


**Causal direction corresponds to shorter description of machine translation data in terms of minimum description length (MDL):**

Data ( $X \rightarrow Y$ )	MDL(X)	MDL(Y)	MDL(Y X)	MDL(X Y)	MDL(X)+MDL(Y X) vs. MDL(Y)+MDL(X Y)
En $\rightarrow$ Es	46.54	105.99	2033.95	2320.93	2080.49 < 2426.92
Es $\rightarrow$ En	113.42	55.79	3289.99	3534.09	3403.41 < 3589.88
En $\rightarrow$ Fr	20.54	53.83	503.78	535.88	524.32 < 589.71
Fr $\rightarrow$ En	53.83	21.6	705.28	681.12	759.11 > 702.72
Es $\rightarrow$ Fr	58.26	55.66	701.04	755.5	759.30 < 811.16
Fr $\rightarrow$ Es	56.14	54.34	665.26	706.53	721.40 < 760.87

# ICM in NLP: Findings

(with Zhijing Jin & Julius von Kügelgen)



**Implications of ICM for SSL and DA confirmed by NLP meta-study:**

Semi-supervised learning (SSL): *anticausal* > *causal*.

<b>Task Type</b>	<b>Mean <math>\Delta</math>SSL (<math>\pm</math>std)</b>	<b>According to ICM</b>
Causal	+0.04 ( $\pm$ 4.23)	Smaller or none
Anticausal	+1.70 ( $\pm$ 2.05)	Larger

Domain adaptation (DA): *causal* > *anticausal*.

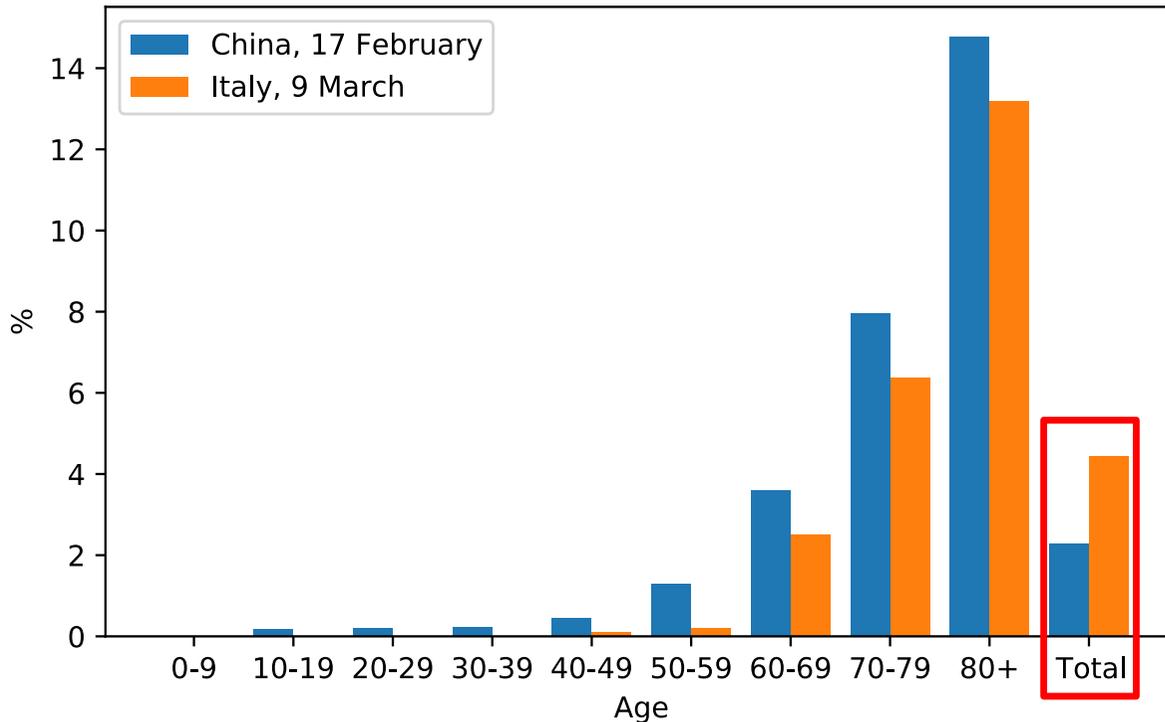
<b>Task Type</b>	<b>Mean <math>\Delta</math>DA (<math>\pm</math>std)</b>	<b>According to ICM</b>
Causal	5.18 ( $\pm$ 6.57)	Larger
Anticausal	1.26 ( $\pm$ 1.79)	Smaller

# Simpson's paradox in Covid-19 case fatality

(v. Kugelgen, Gresele, <https://arxiv.org/abs/2005.07180> / IEEE Trans. AI)



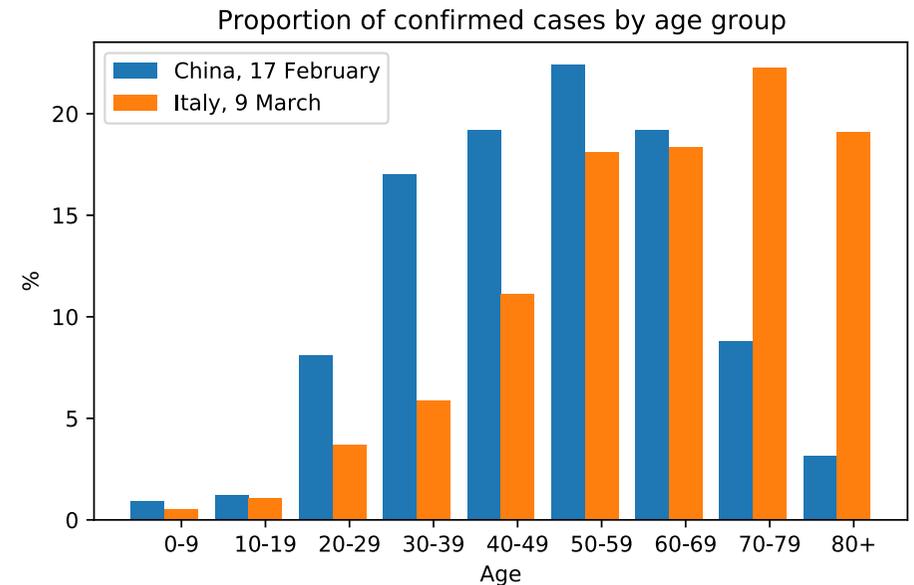
Case fatality rates (CFRs) by age group



Case fatality rates (CFRs) in Italy are *lower* for each age group, but *higher* overall.

**Simpson's paradox:** opposite trends in grouped and aggregated data.

Here, it stems from a difference in case demographic:



Thanks to Elias Bareinboim

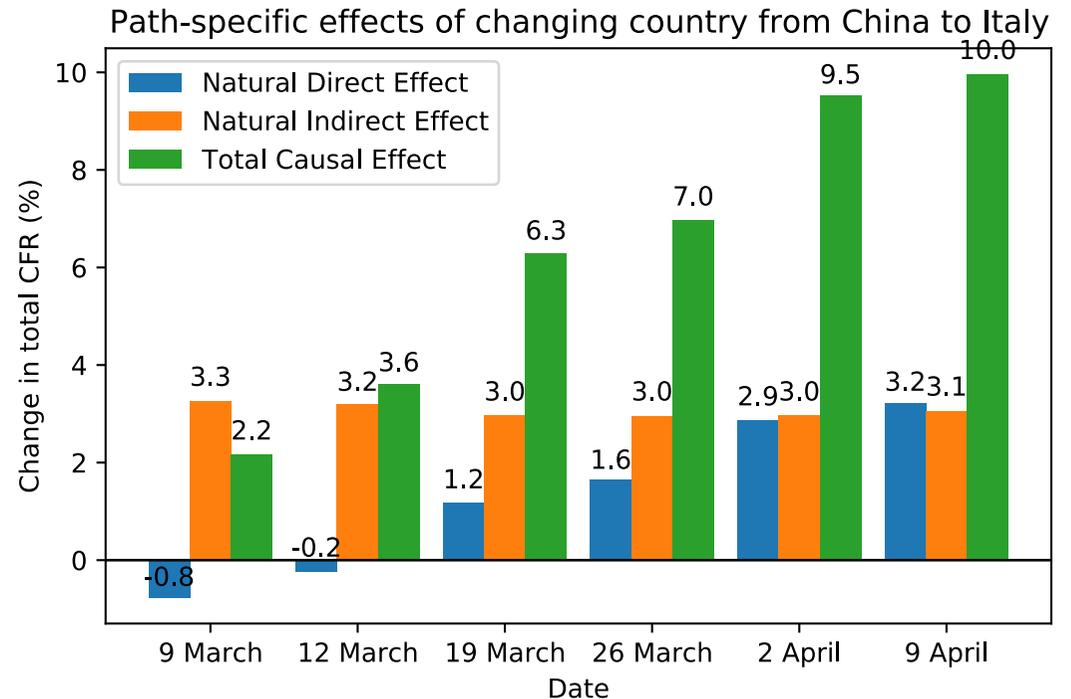
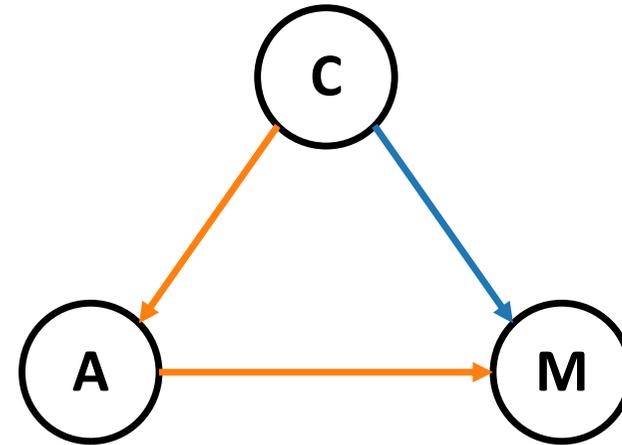
# Mediation analysis

Only for linear models can **total causal effect (TCE)** be decomposed into direct effect (DE) and indirect effect (IE),

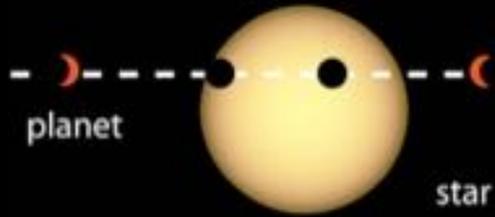
$$\text{TCE} = \text{DE} + \text{IE}$$

Due to interactions, DE and IE are *not uniquely defined in general*, but depend on the state of the mediator.

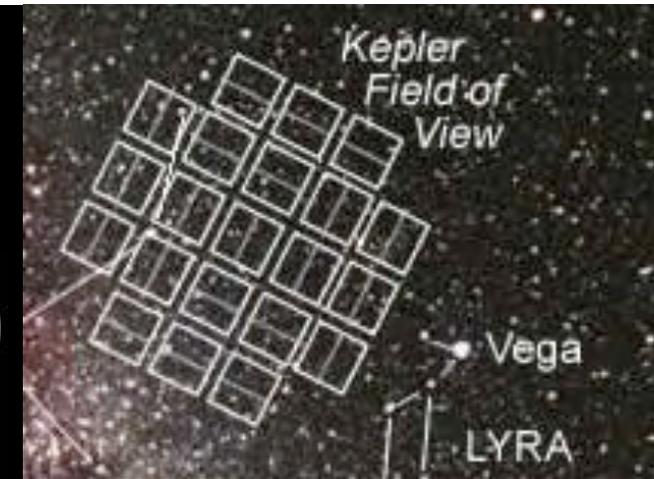
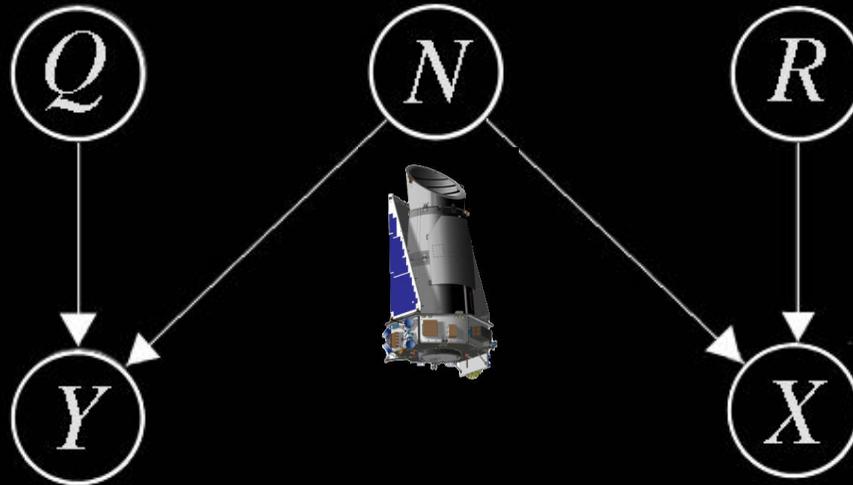
- **Natural Direct Effect (NDE):** case demographic kept as in China while CFRs per age group changed to those in Italy.
- **Natural Indirect Effect (NIE):** CFRs per age group kept as in China, while case demographic changed to that in Italy.



# Causal models for exoplanet detection

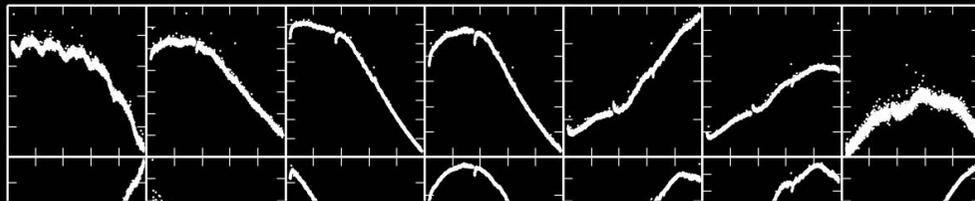


ICML 2015  
Astrophysical Journal 2015  
PNAS 2016

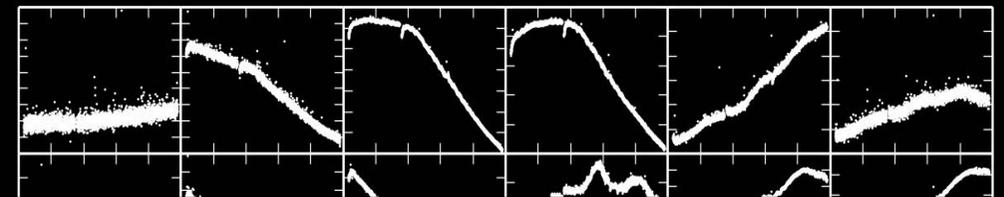


$$Q - E[Q] = Y - E[Y|X]$$

Kepler 5088536 Quarter 5  
CCD channel 25 Row 875 Column 322



Kepler 5949551 Quarter 5  
CCD channel 25 Row 57 Column 756



Discovered 21 new exoplanets. One of them received the name K2-18b.

# The Milkman Problem



# Water found on a potentially life-friendly alien planet

A super-Earth about 111 light-years away is “the best candidate for habitability that we know right now,” astronomers say.



3 MINUTE READ



## Water found on most habitable known world beyond solar system

But humans would not fare well on planet K2-18b despite wispy clouds and huge red sun



Astronomische Sensation

### Wasserdampf auf Planet K2-18b



### Observations

## No, the Exoplanet K2-18b Is *Not* Habitable

News outlets that said otherwise are just crying wolf—but they’re not the only ones at fault

By Laura Kreidberg on September 23, 2019

Missions | Galleries | NASA TV | Follow

Humans in Space | Moon to Mars | Earth | Space Tech | Flight | Science

UNDARK

Opinion: Exoplanets, Life, and the Danger of a Single Study

There's value in covering new research advances, even when the science is still unsettled. But there are also risks.

Viewed: ESA | Hubble | M. Kornmesser

Hubble

NASA's Hubble Finds Water Vapor on Habitable-Zone Exoplanet for 1st Time

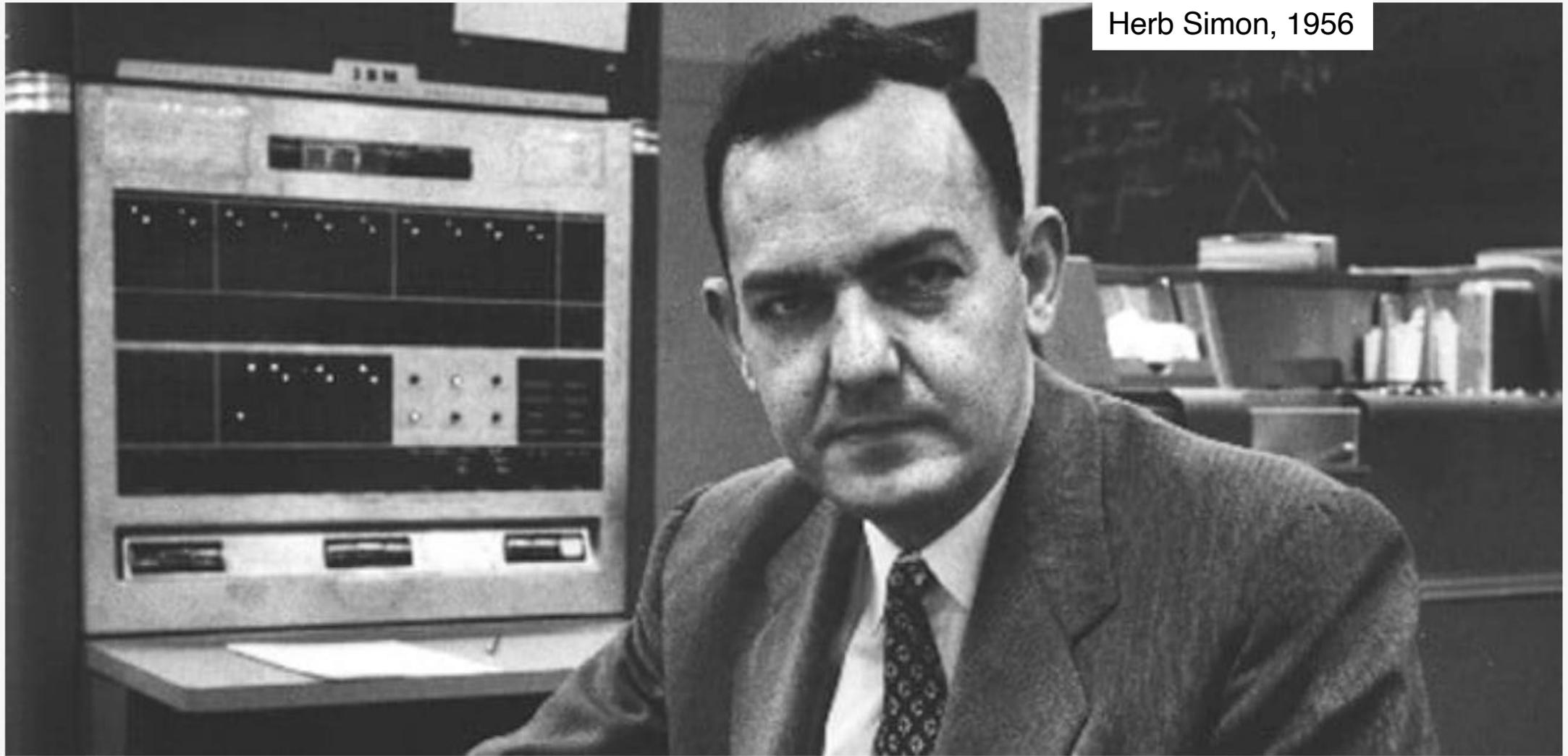
Credits: ESA/Hubble, M. Kornmesser

*Bernhard Schölkopf*



MAX-PLANCK-GESELLSCHAFT

Herb Simon, 1956



**“Machines will be capable, within twenty years, of doing any work a man can do”**

## Toward causal representation learning

**Core Problem of Statistical Representations:** Representation learning only includes *statistical* information — it does not capture interventions, reasoning, planning.

**Core Problem of Causal Representations:** SCMs are usually at the *symbolic* level — they assume the causal variables are given.

<https://arxiv.org/abs/2102.11107>

# Independent mechanisms and the disentangled factorization

## Factorization

- independent noises in the causal graph:

$$p(X_1, \dots, X_n) = \prod_{I=1}^n p(X_i | PA_i)$$

# Independent mechanisms and the disentangled factorization

## Disentangled (causal) factorization

<https://arxiv.org/abs/1911.10500>

<https://arxiv.org/abs/2102.11107>

- independent noises in the causal graph:

$$p(X_1, \dots, X_n) = \prod_{I=1}^n p(X_i | PA_i)$$

- independent mechanisms: changing one  $p(X_i | PA_i)$  does not change the other  $p(X_j | PA_j)$  ( $j \neq i$ ); they remain **invariant**

(Janzing & Schölkopf, *IEEE Trans. Inf. Th.* 2010; Schölkopf et al., *ICML* 2012),

cf. *autonomy, (structural) invariance, separability, exogeneity, stability, modularity*: (Aldrich, 1989; Pearl, 2009)

Special case: If the graph has no edges, disentanglement reduces to statistical independence:

$$p(X_1, \dots, X_n) = \prod_{I=1}^n p(X_i)$$

In general, the causal factors will not be statistically independent, and independence-based methods struggle to find them (Träuble et al., *ICML* 2021)

# Entangled factorizations

## Disentangled (causal) factorization

$$p(X_1, \dots, X_n) = \prod_{I=1}^n p(X_i \mid \text{PA}_i)$$

## Entangled (non-causal) factorizations

e.g.,

$$p(X_1, \dots, X_n) = \prod_{I=1}^n p(X_i \mid X_{i+1}, \dots, X_n).$$

- cannot intervene on  $p(X_i \mid X_{i+1}, \dots, X_n)$
- changing one  $p(X_i \mid \text{PA}_i)$  will usually change **many** of the  $p(X_i \mid X_{i+1}, \dots, X_n)$

# Causal viewpoint on distribution shift

Disentangled causal factorization

$$p(X_1, \dots, X_n) = \prod_{I=1}^n p(X_i | PA_i)$$

with independent mechanisms  $p(X_i | PA_i)$ .

**Sparse Mechanism Shift Hypothesis:** small distribution changes manifest themselves sparsely in the disentangled factorization, i.e., they should usually not affect all factors simultaneously.

Here, a shift can be passive (e.g., distribution drift) or active (intervention, action).

Stated in (Parascandolo et al., arXiv:1712.00961 (2017); Bengio et al., arXiv:1901.10912 (2019), Schölkopf, arXiv:1911.10500 (2019)); see also (Schölkopf et al., ICML 2012, Schölkopf, Janzing, Lopez-Paz 2016, Zhang et al., ICML 2013, Huang, Zhang et al., JMLR 2020)

# Causal training

**Sparse mechanism shift training:** require that across domain shifts or actions/interventions, only a sparse set of causal representation factors changes.

**ICM training:** encourage independence of mechanisms

**Counterfactual training:** require that interventions produce valid images (e.g., after reconstruction in an autoencoder).

For *interventions on the  $U_i$* , this encourages statistical independence (cf. standard disentanglement).

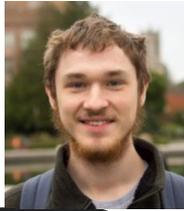
For *interventions on the  $S_i$* , this encourages the mechanisms  $f_i$  to be independently manipulable.

**Structural training:** embed SCM structure into decoder architecture and train by reconstruction error

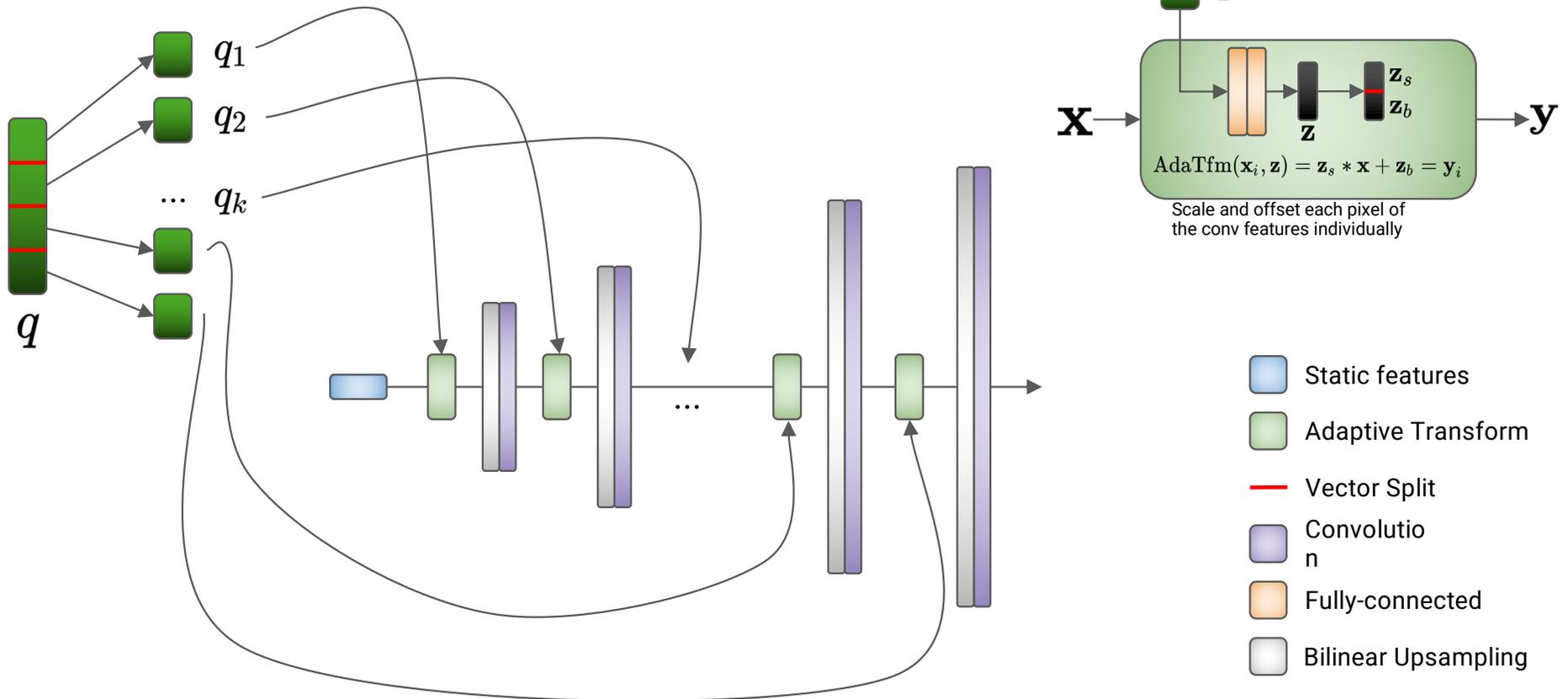
# Structural Decoders

Leeb et al.

*arXiv 2006.07796*



→ ~200-700k parameters



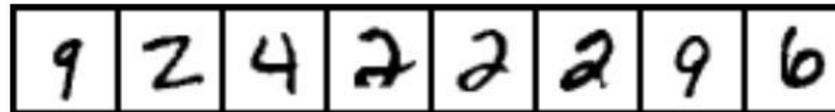


# Learning independent mechanisms

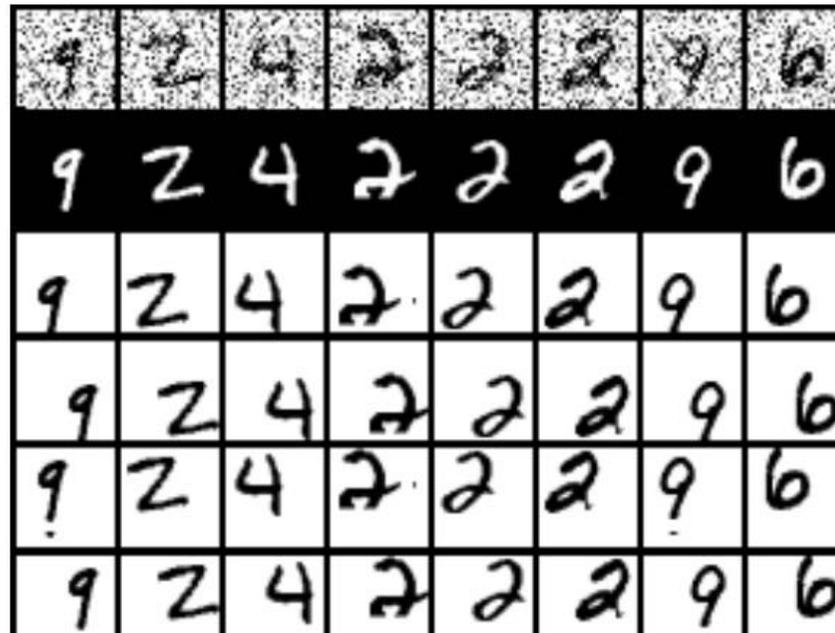
(with Parascandolo, Kilbertus, Rojas-Carulla, ICML 2018)



- Data drawn from  $p(x)$ , transformed by  $M$  mechanisms  $f_1, \dots, f_M$
- Goal: learn the independent mechanisms / factors of variation
- Method: generative model with competing mechanisms



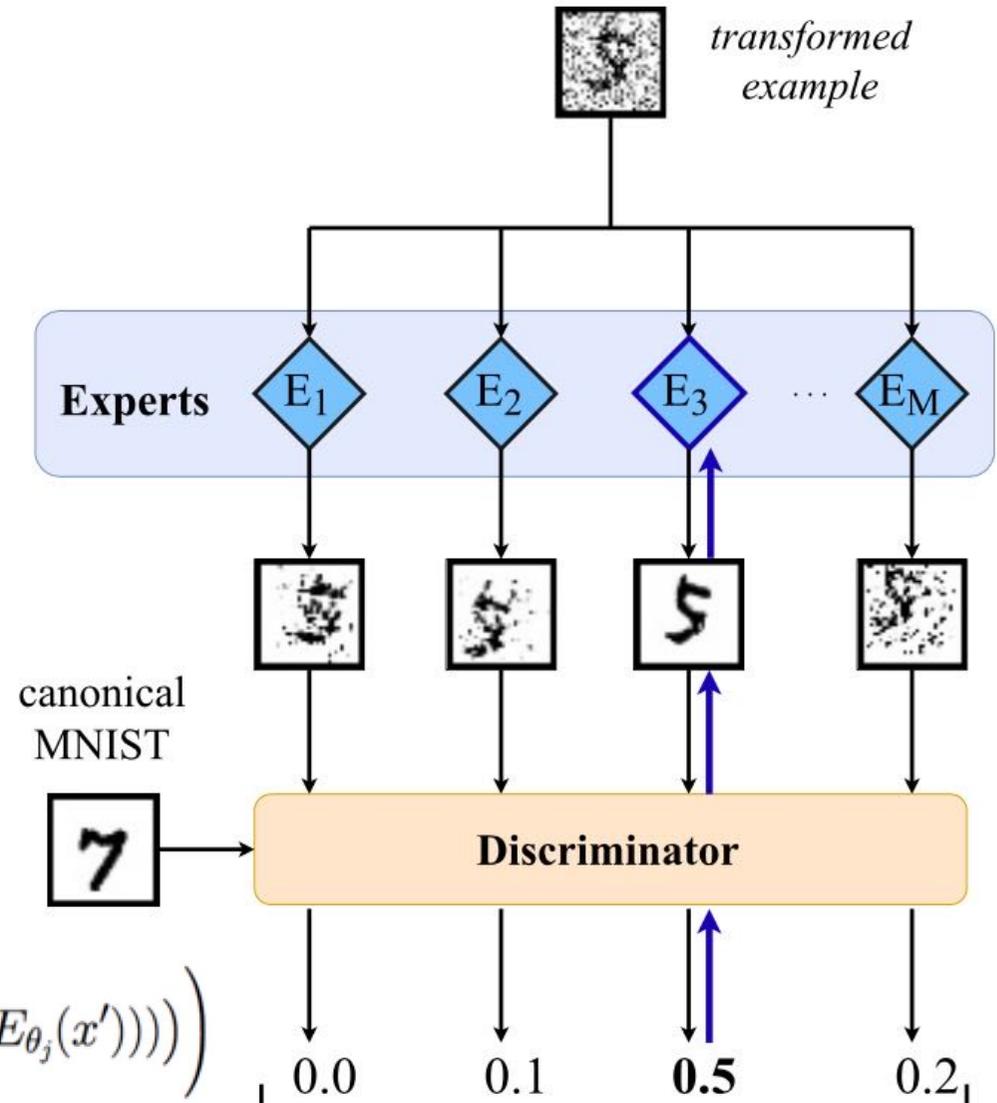
Original data



Transformed data

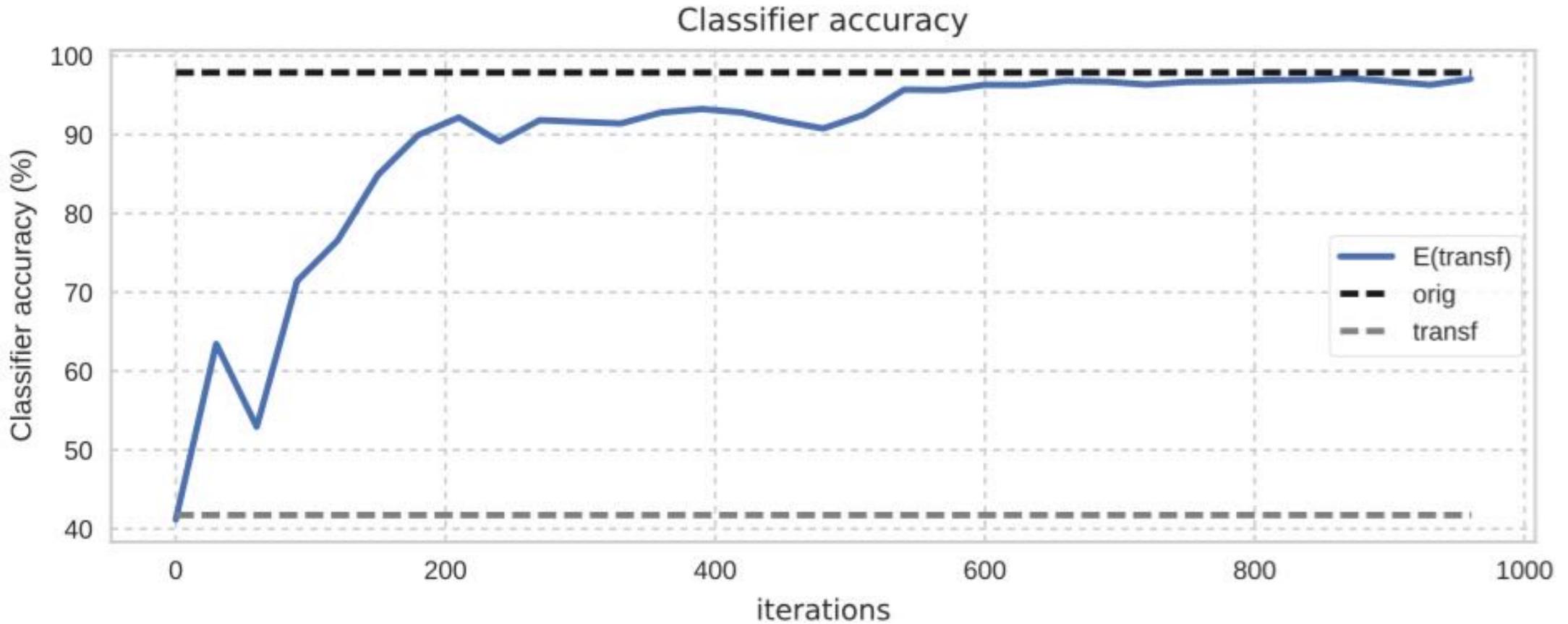
## Method

- Mechanisms initialized  $\approx$  identity
- The highest scoring mechanism against the discriminator  $D$  wins the example and is updated to increase the score
- $D$  is trained on the original data and against the winning outputs



$$\max_{\theta_D} \left( \mathbb{E}_{x \sim P} \log(D_{\theta_D}(x)) + \frac{1}{N'} \sum_{j=1}^{N'} \mathbb{E}_{x' \sim Q} (\log(1 - D_{\theta_D}(E_{\theta_j}(x')))) \right)$$

# Accuracy of a CNN trained on MNIST for different test sets

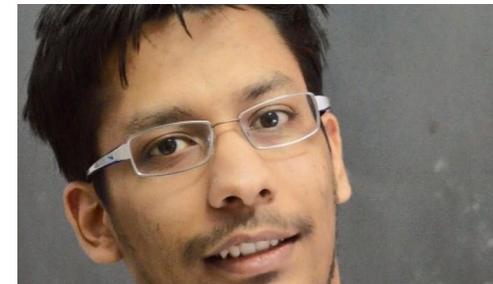


# Generalizing to Omniglot characters

Inputs

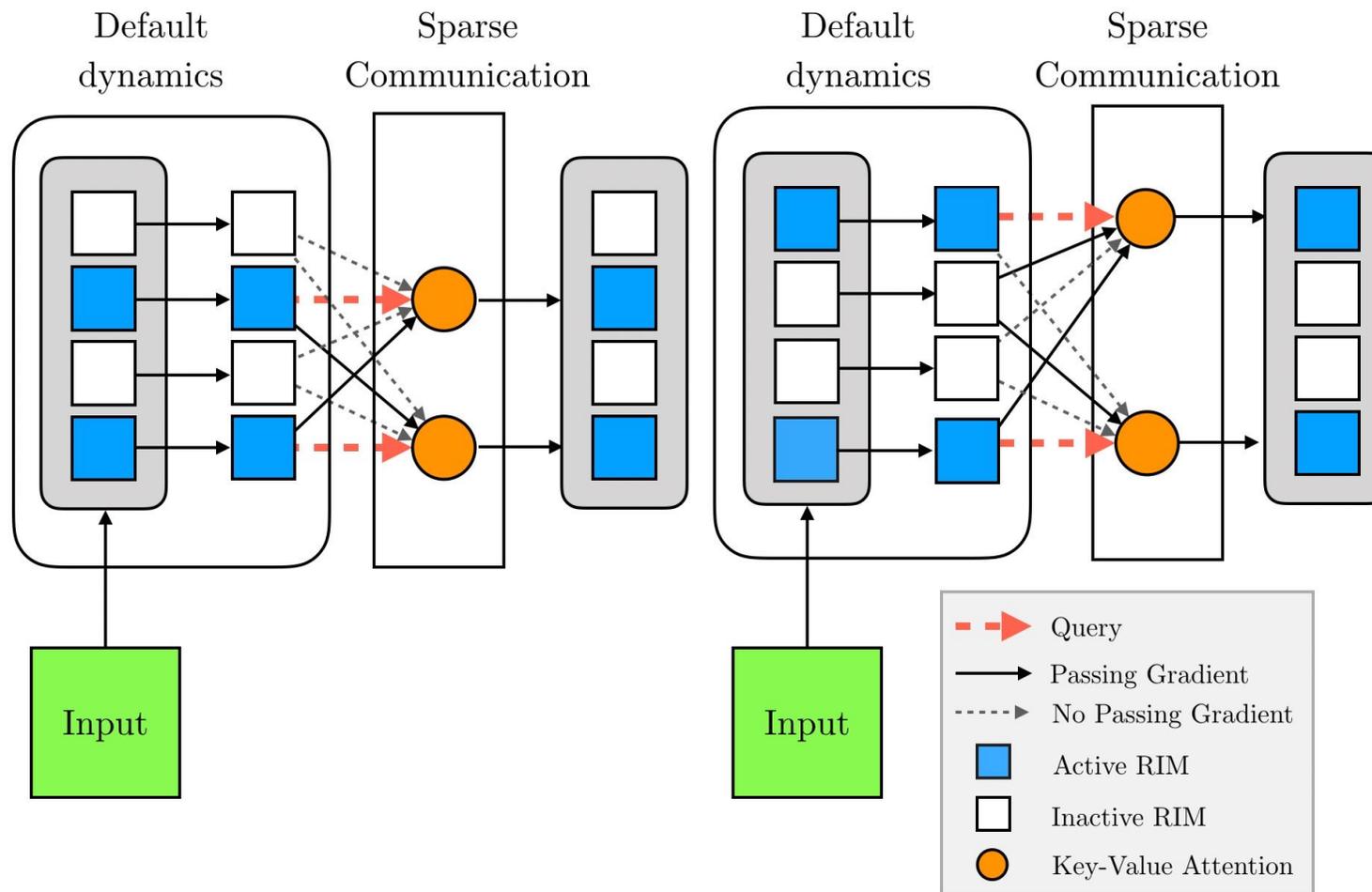
Exp0									
Exp1									
Exp2									
Exp3									
Exp4									
Exp5									
Exp6									
Exp7									
Exp8									
Exp9									

# Recurrent Independent Mechanisms



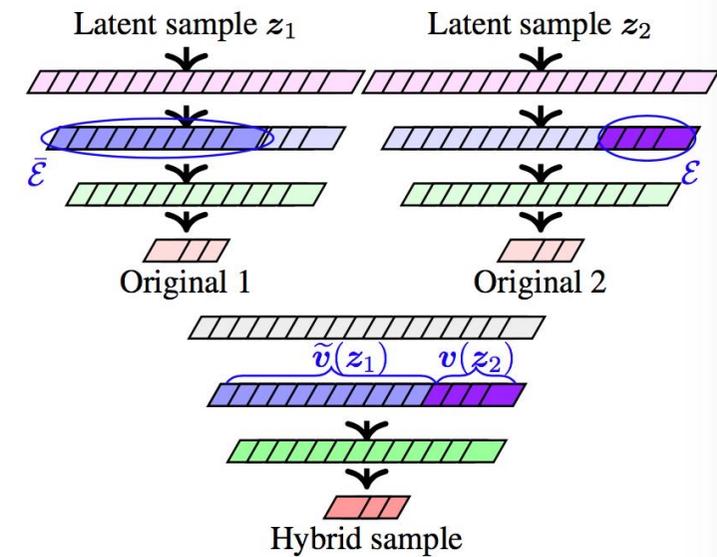
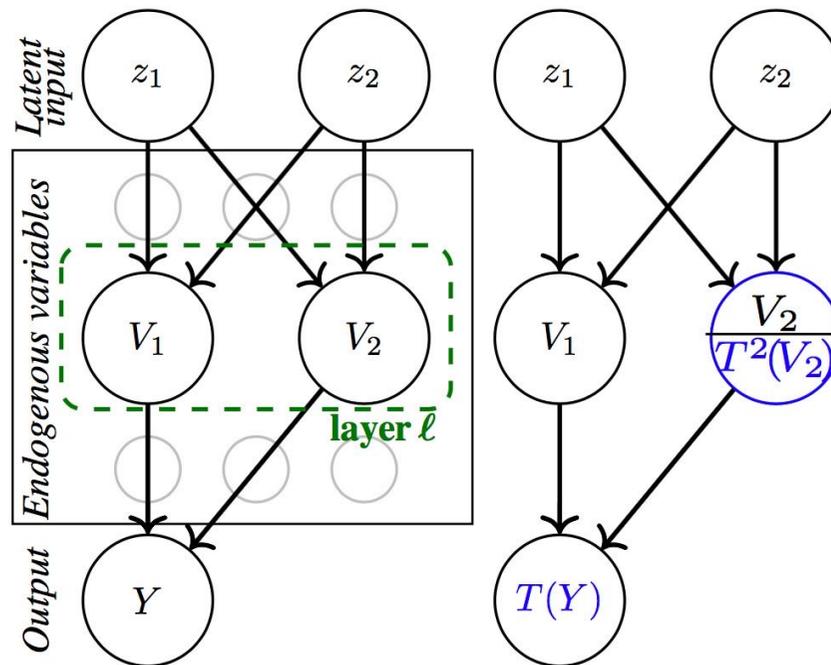
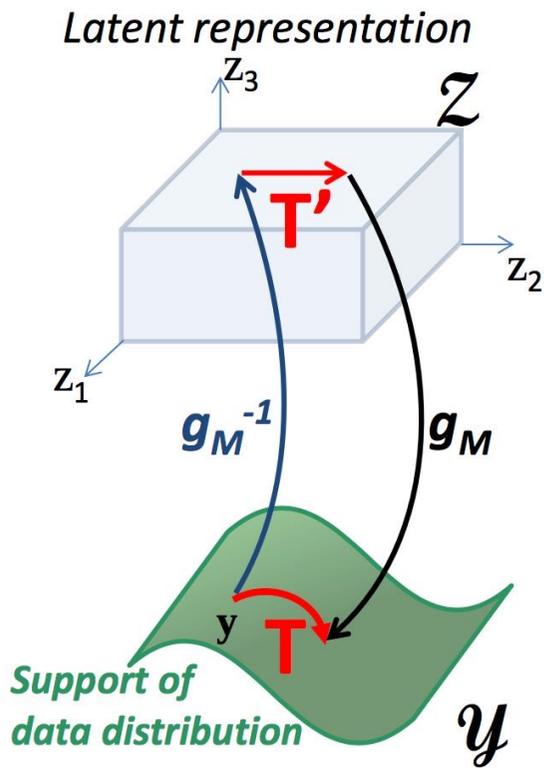
with **Anirudh Goyal**,  
Alex Lamb,  
Jordan Hoffmann,  
Shagun Sodhani,  
Sergey Levine,  
Yoshua Bengio

ICLR 2021



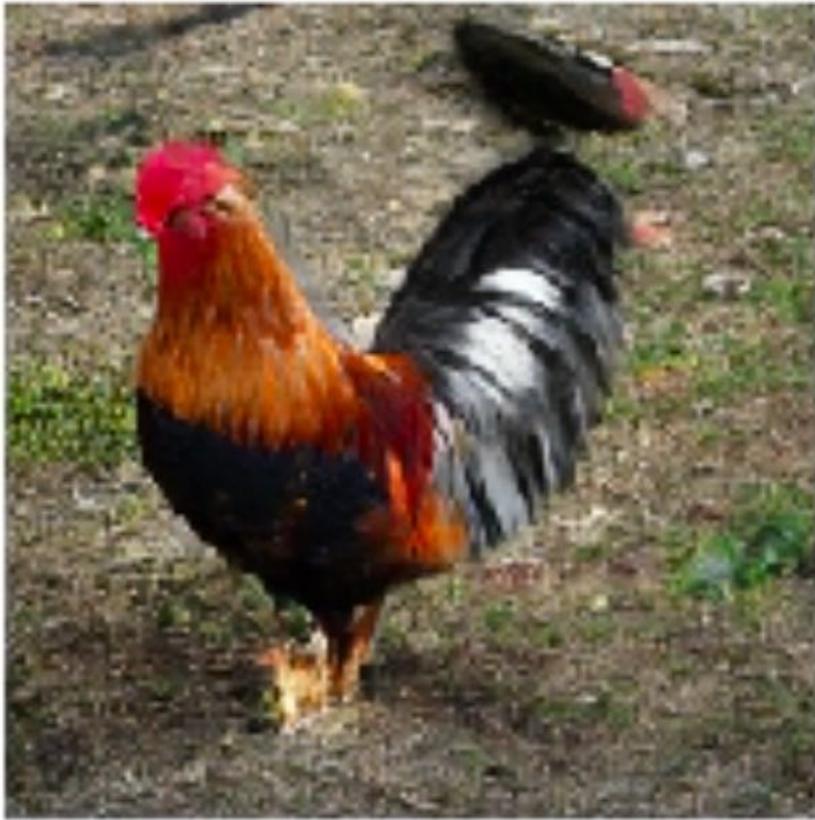
A. Goyal, A. Lamb, J. Hoffmann, S. Sodhani, S. Levine, Y. Bengio, and B. Schölkopf, 2019. Recurrent independent mechanisms. [arXiv:1909.10893](https://arxiv.org/abs/1909.10893).

# Interventional Representations (Besserve et al., ICLR 2020)



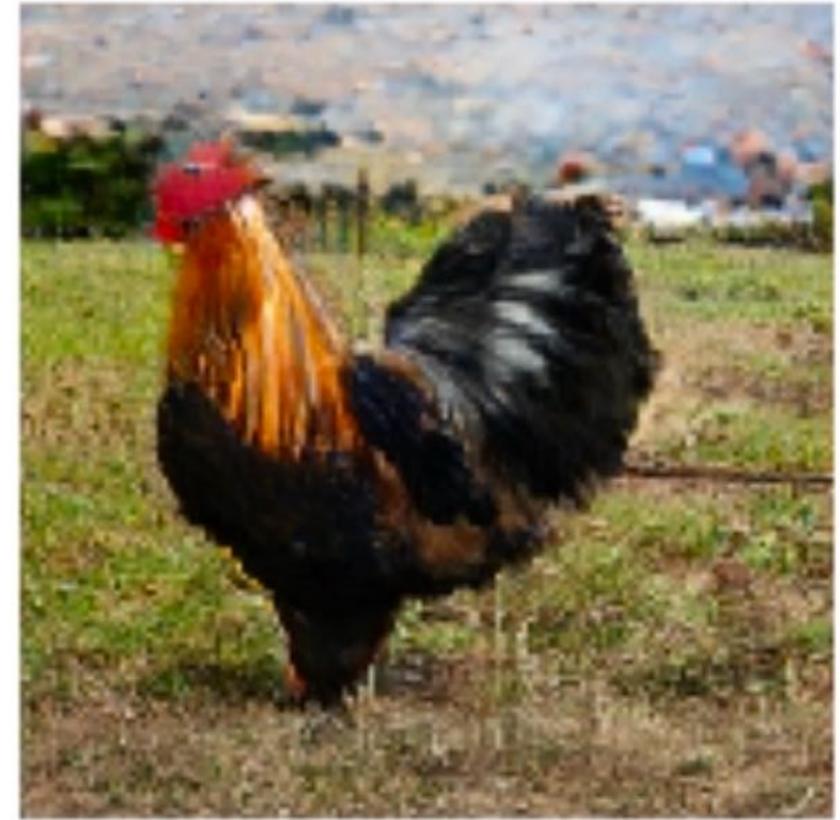
# Interventional Representations *(Besserve et al., ICLR 2020)*

**Original**



Causal  
Intervention

**Counterfactual**



# Interventional Representations *(Besserve et al., ICLR 2020)*

Original 1



Hybrid



Original 2

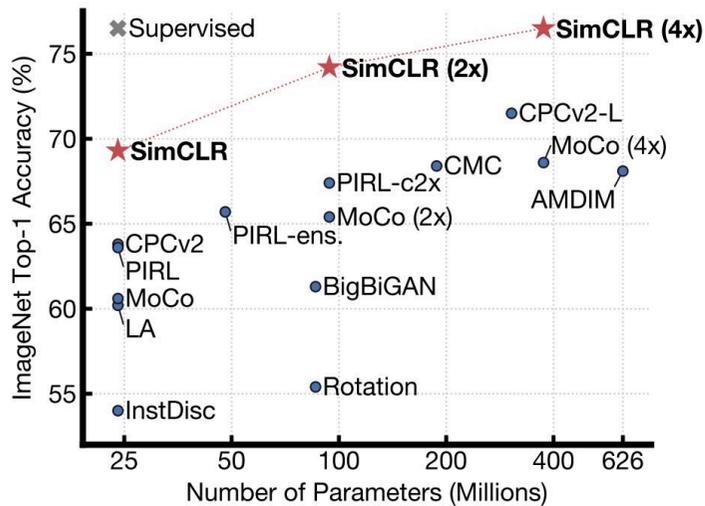


# Self-supervised learning provably isolates content from style

(<https://arxiv.org/abs/2106.04619>)

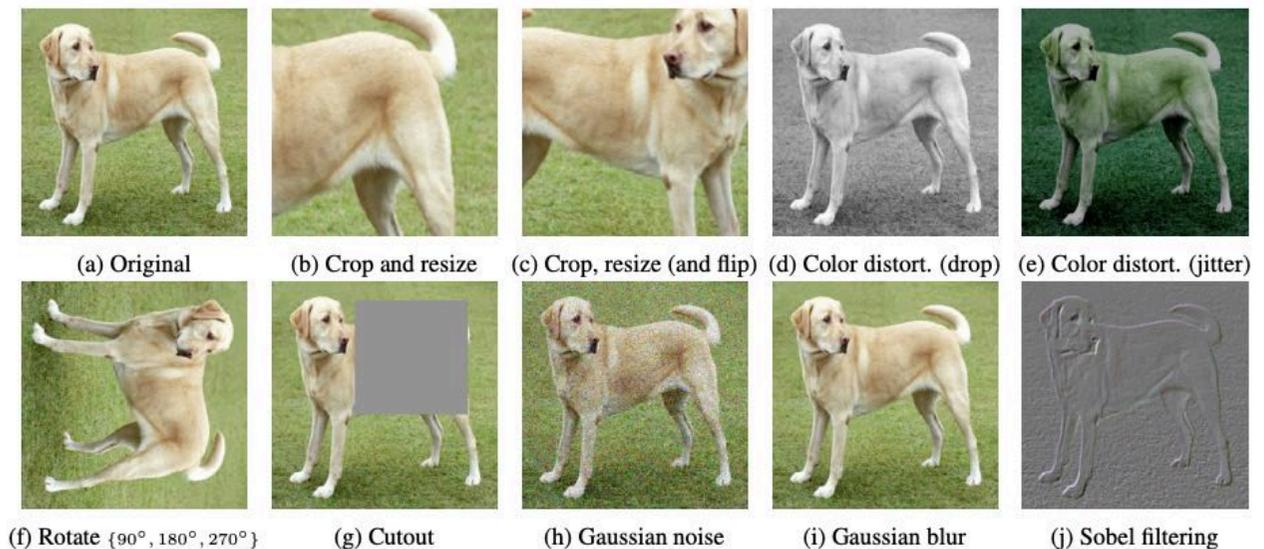


with **Julius von Kügelgen\***,  
**Yash Sharma\***, **Luigi Gresele\***,  
 Wieland Brendel, Michel  
 Besserve, Francesco Locatello



Data augmentation: very effective for self-supervised representation learning.

Useful because transformations are typically designed to leave *semantics* intact:



Figures from:

*SimCLR: A Simple Framework for Contrastive Learning of Visual Representations.*  
 Chen, Kornblith, Norouzi, Hinton (ICML 2020; <https://arxiv.org/abs/2002.05709>)

# Self-supervised learning with data augmentations provably isolates content from style



with Julius von Kügelgen\*,  
Yash Sharma\*, Luigi Gresele\*,  
Wieland Brendel, Michel  
Besserve, Francesco Locatello

Formalise generation  $x = f(z)$  and augmentation  $\tilde{x} = f(\tilde{z})$  processes as latent variable model with a content-style partition  $z = (c, s)$ :

- *invariant content  $c$* : always shared between pairs  $(x, \tilde{x})$  of views;
- *varying style  $s$* : may change across pairs  $(x, \tilde{x})$  of views.

Allow causal dependence of style on content (*Causal3DIdent* dataset):

augmented view  $\tilde{x}$  = counterfactual under soft style intervention on  $x$ .

**Theory:** Can identify\* invariant content partition in generative and discriminative learning with entropy maximisation (e.g., SimCLR).

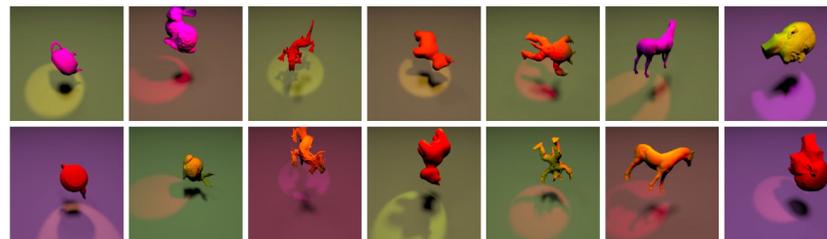
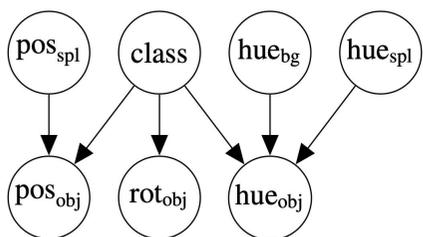
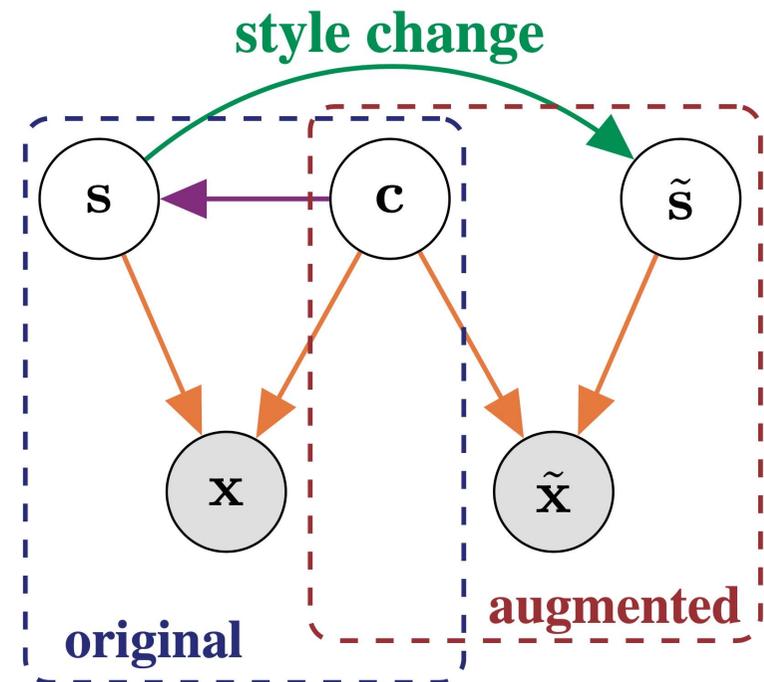


Figure 2: (Left) Causal graph for the *Causal3DIdent* dataset. (Right) Two samples from each object class.



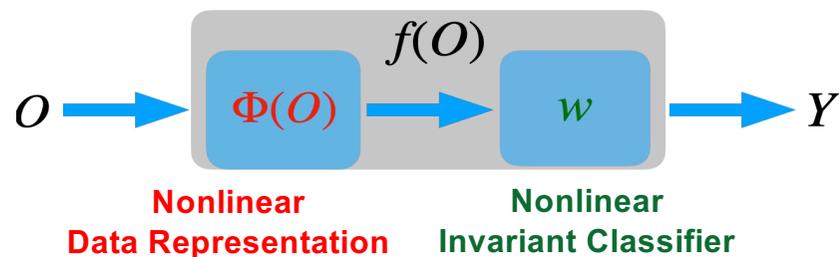
\*up to invertible transformation

# Nonlinear Invariant Risk Minimization

(with Chaochao Lu, Yuhuai Wu, José Miguel Hernández-Lobato, [arXiv:2102.12353](https://arxiv.org/abs/2102.12353))

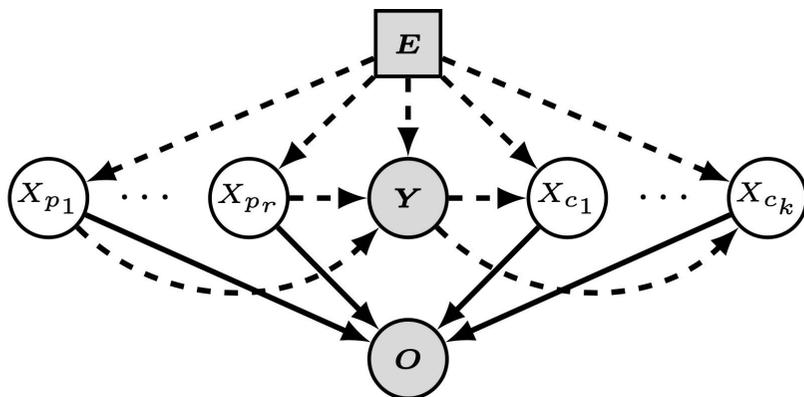


## Problem



**Key Idea:**  
**Data representation  $\Phi(O)$  should be the direct cause of  $Y$ .**

## Assumption on Causal Graphs



This assumption is **more general** than the **common Independence assumption** in latent variable models.

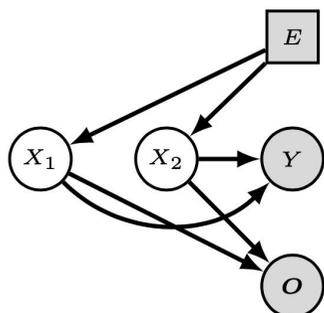
## Assumption on the Prior

$$P(X|Y, E) = P(X_{p_1}, \dots, X_{p_r} | Y, E) \prod_{i \in I_C} P(X_i | Y, E)$$
$$p_{T, \lambda}(X | Y, E) = \frac{Q(X)}{\mathcal{Z}(Y, E)} \cdot \exp(\langle T(X), \lambda(Y, E) \rangle)$$

The prior is assumed to be a **general exponential family distribution** leading to **IDENTIFIABILITY**.

# Experimental Results

(with Chaochao Lu, Yuhuai Wu, José Miguel Hernández-Lobato, [arXiv:2102.12353](https://arxiv.org/abs/2102.12353))



Data Generating Process

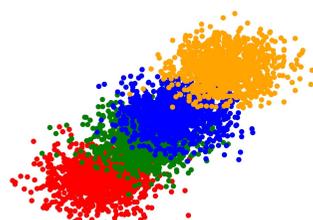
$$E \sim \mathcal{U}\{0.2, 2, 3, 5\}$$

$$X_1 \sim \mathcal{N}(X_1|E, 1)$$

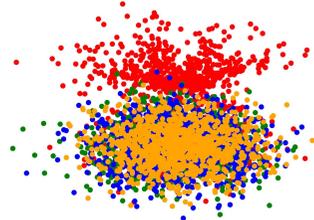
$$X_2 \sim \mathcal{N}(X_2|2E, 2)$$

$$Y \sim \mathcal{N}(Y|X_1 + X_2, 1)$$

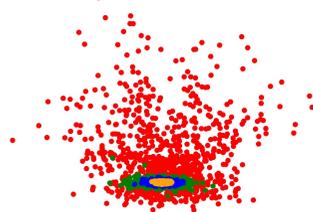
$$O = g(X_1, X_2)$$



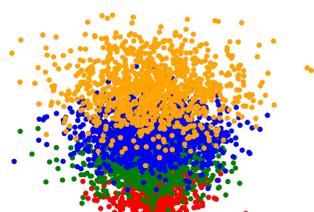
Original Data



Samples from VAE  
(Kingma et al. 2013)



Samples from iVAE  
(Khemakhem et al. 2020)



Samples from iCaRL



Color	Red	Green
Y=0	$p_e$	$1 - p_e$
Y=1	$1 - p_e$	$p_e$

- **2 Training Envs:**  
 $\{p_e = 0.1, p_e = 0.2\}$
- **1 Testing Env:**  
 $\{p_e = 0.9\}$

Table 2: Colored Fashion MNIST. Comparisons in terms of accuracy (%) (mean  $\pm$  std deviation).

METHOD	TRAIN	TEST
ERM	$83.17 \pm 1.01$	$22.46 \pm 0.68$
ERM 1	$81.33 \pm 1.35$	$33.34 \pm 8.85$
ERM 2	$84.39 \pm 1.89$	$13.16 \pm 0.82$
ROBUST MIN MAX	$82.81 \pm 0.11$	$29.22 \pm 8.56$
F-IRM GAME	$62.31 \pm 2.35$	$69.25 \pm 5.82$
V-IRM GAME	$68.96 \pm 0.95$	$70.19 \pm 1.47$
IRM	$75.01 \pm 0.25$	$55.25 \pm 12.42$
<b>iCaRL (ours)</b>	<b><math>74.87 \pm 0.36</math></b>	<b><math>73.56 \pm 0.75</math></b>
ERM GRAYSCALE	$74.79 \pm 0.37$	$74.67 \pm 0.48$
OPTIMAL	75	75

# Source-Free Adaptation to Measurement Shift via Bottom-Up Feature Restoration

(Cian Eastwood et al., <https://arxiv.org/abs/2107.05446>)



## Source-free domain adaptation

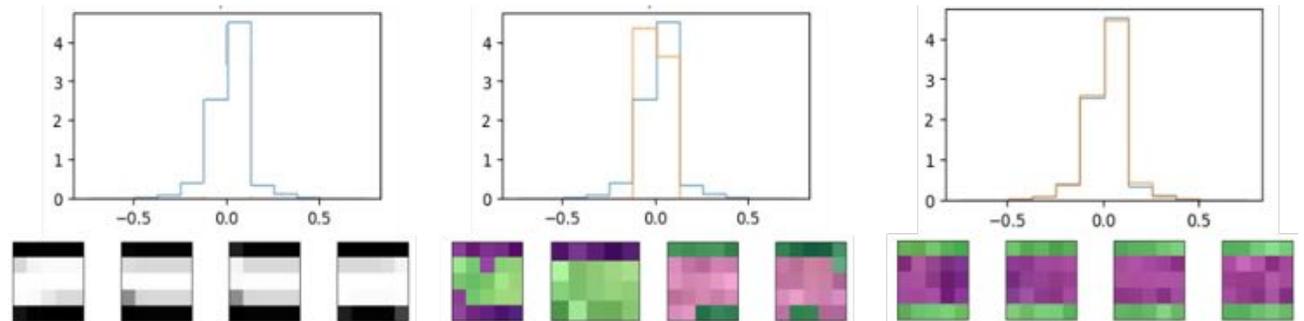
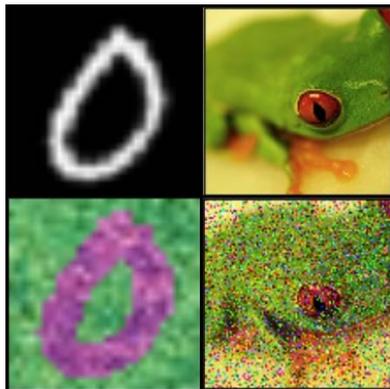
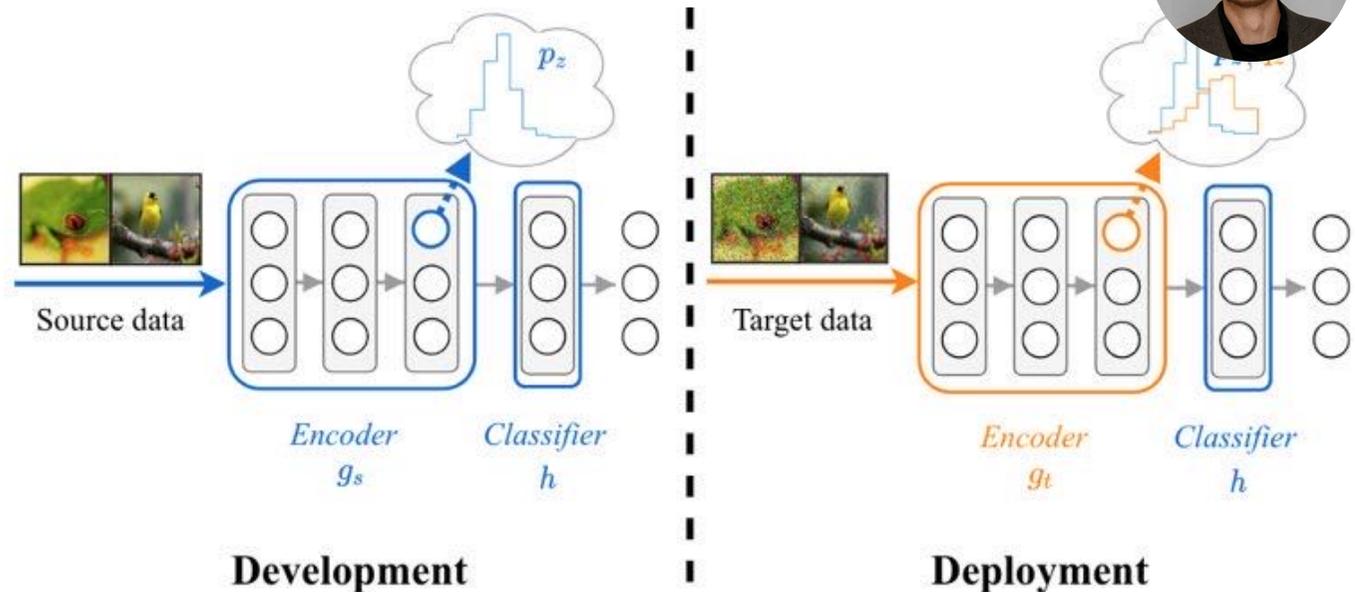
- Development: train + equip model
- Deployment: adapt, no source data

## Measurement shift (cf. Storkey, 2009)

- New sensor, same underlying features

## Feature restoration

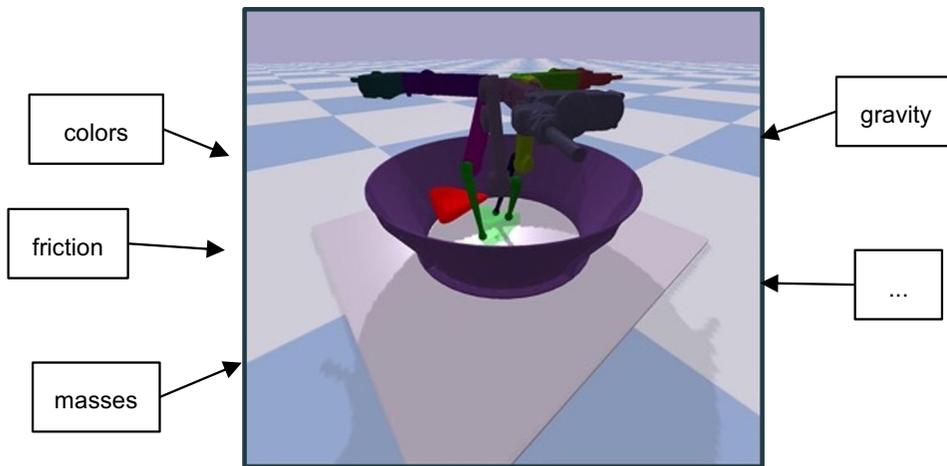
- Goal: extract same features, new env.
- Method: align (marginal) feature dists.



# CausalWorld: A Robotic Manipulation Benchmark for Causal Structure and Transfer Learning

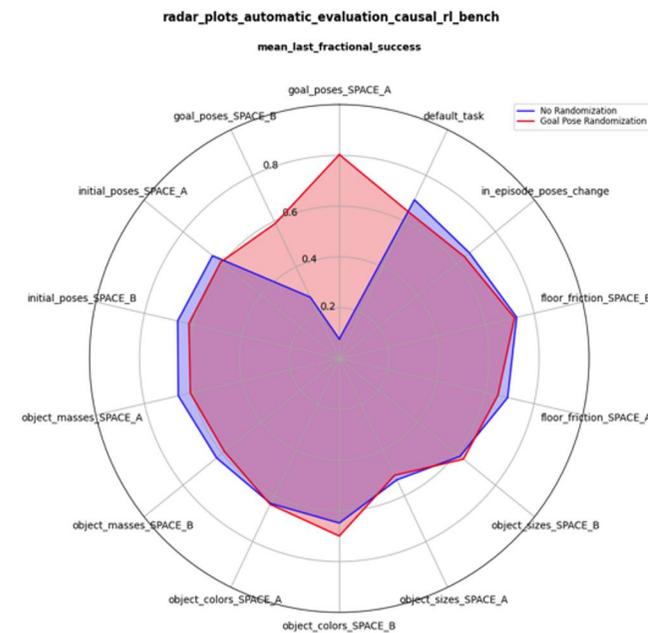


Ahmed and Träuble et al.,  
arXiv: 2010.04296,  
ICLR 2021



**Evaluate different generalization aspects** by intervening on a large range of different defining variables of the hierarchical causal generative world model of the robotic environment.

Benchmark with many challenging environments and fully documented code: <https://github.com/rr-learning/CausalWorld>



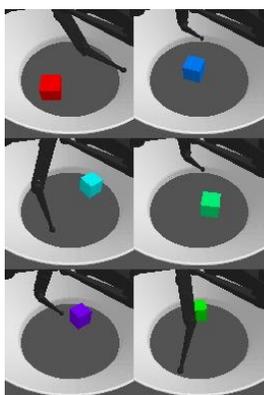
# On the Transfer of Disentangled Representations in Realistic Settings



Dittadi and Träuble et al.,  
arXiv: 2010.14407,  
ICLR 2021

## New Disentanglement Dataset

More complex and realistic, correlations between factors, occlusions, sim-to-real

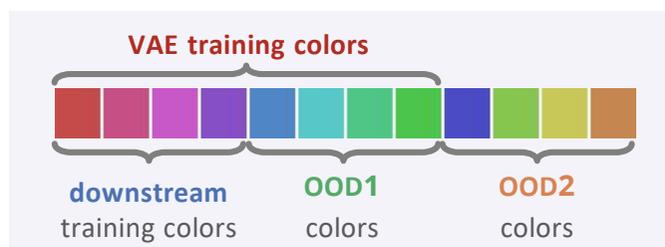


1 million simulated



1800 real (labeled)

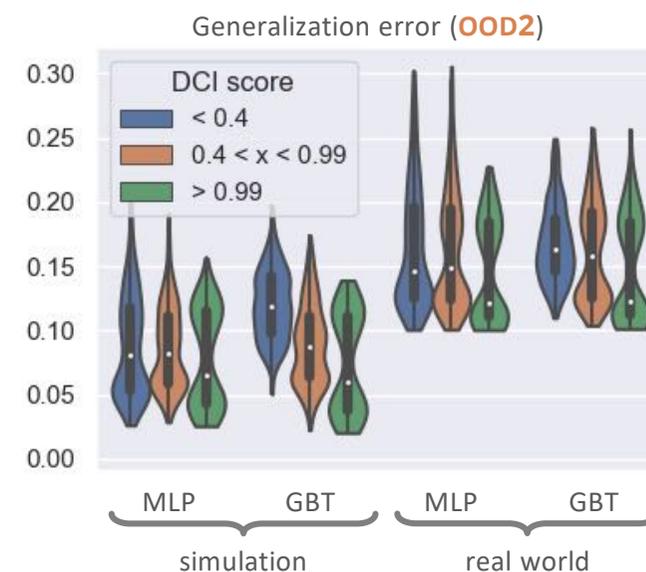
## Out-of-Distribution Generalization of Downstream Tasks



Task: predict value of non-OOD factors

- Train downstream task on pre-trained representations
- Test it OOD but still in the VAE's training distribution (OOD1)
- Test it OOD w.r.t. the VAE itself (OOD2)

Disentanglement has **minor role** when represent. function is OOD



# Causal Curiosity: RL Agents Discovering Self-supervised Experiments for Causal Representation Learning

- Curiosity to discover causation in an environment.
- **Reward-free**
- Set of environments with interventions on causal factors
- Use Kolmogorov Complexity as reward to RL agent
- Agents producing self-supervised experiments to test out mass, size etc.

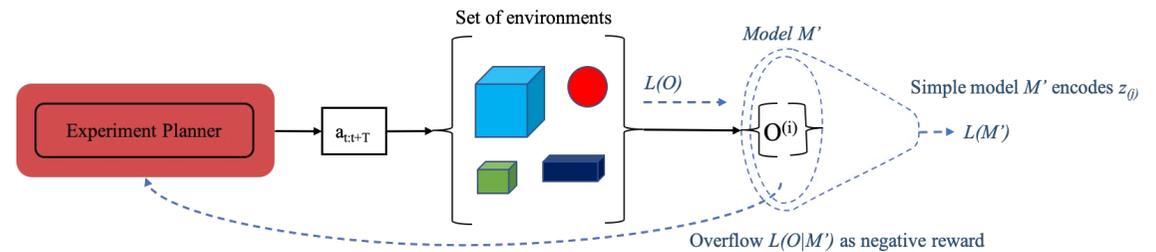


Fig 1: Experiment Discovery

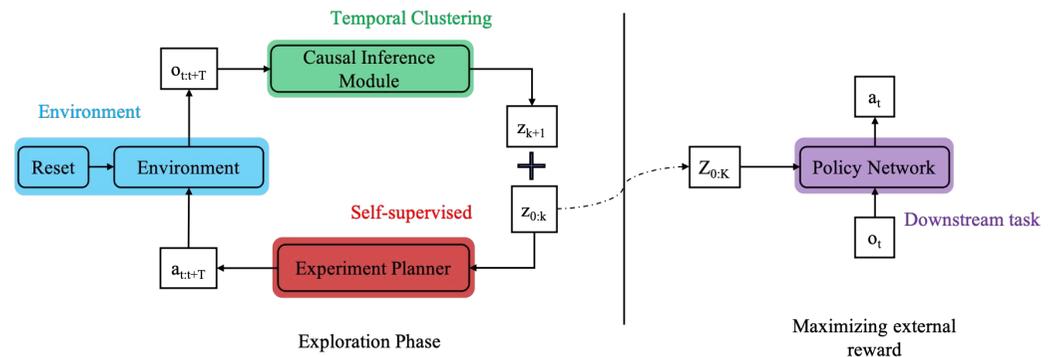
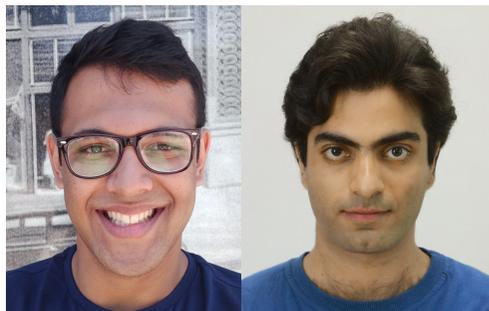


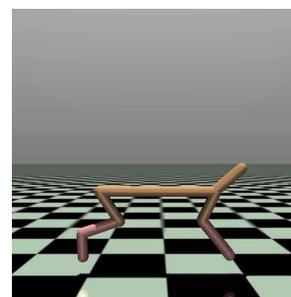
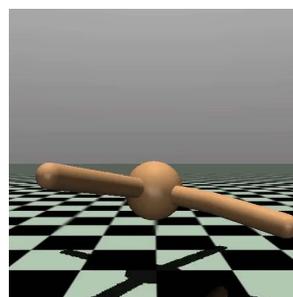
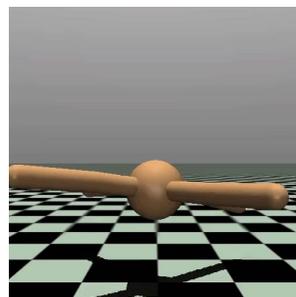
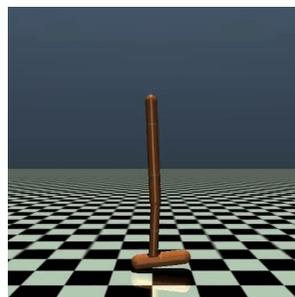
Fig 2: Performing experiments sequentially to learn causal representations. Representations used for downstream transfer.



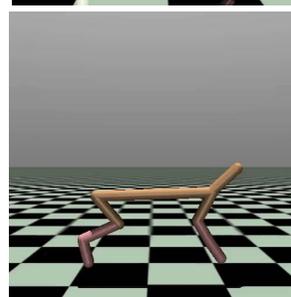
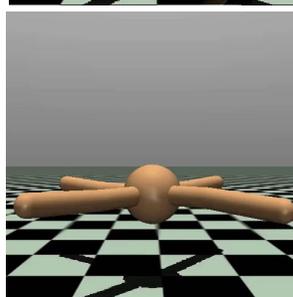
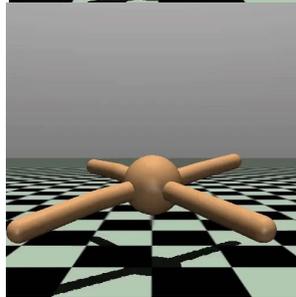
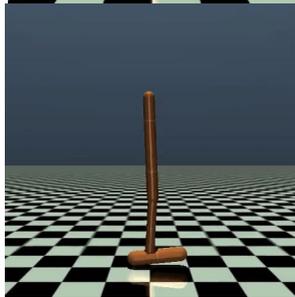
Sontakke, Sumedh A., Arash Mehrjou, Laurent Itti, and Bernhard Schölkopf. "Causal Curiosity: RL Agents Discovering Self-supervised Experiments for Causal Representation Learning." *arXiv preprint arXiv:2010.03110* (2020). To appear at *ICML 2021*

# Discovered Behaviors - Mujoco

Heavy



Light



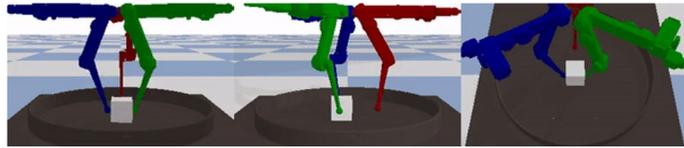
Hop

Pirouette

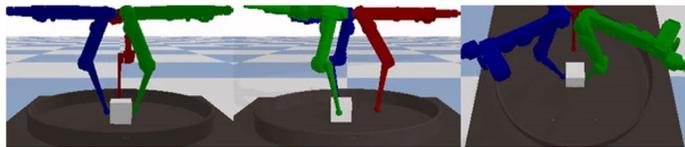
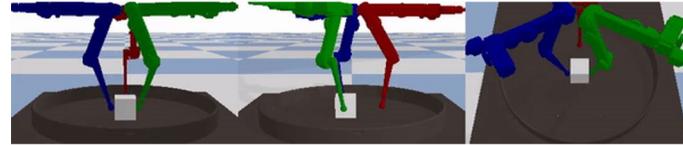
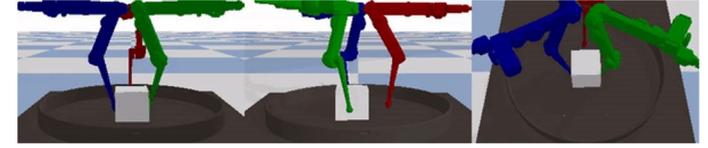
Leap

Pushup

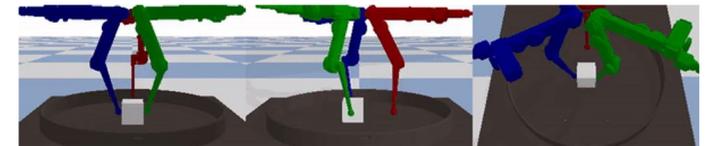
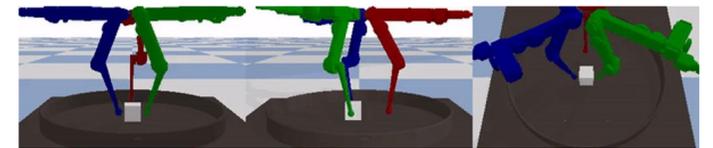
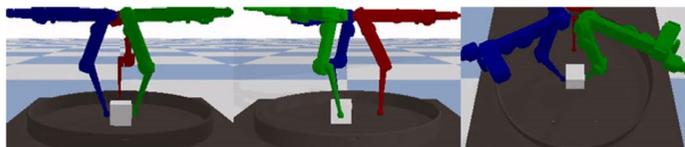
# Discovered Behaviors - CausalWorld



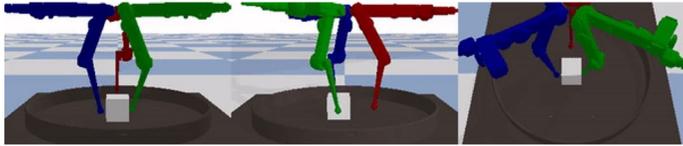
Lifting Behaviors



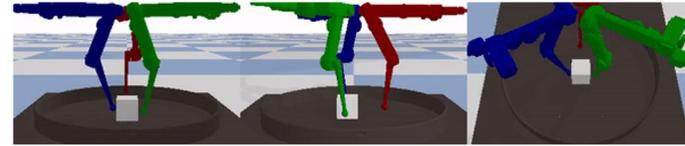
Rotate Behaviors



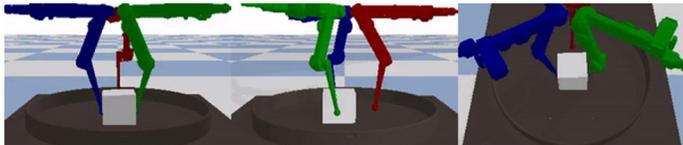
# Discovered Behaviors - CausalWorld



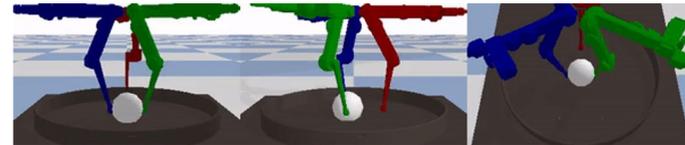
Dribble



Pushing along y



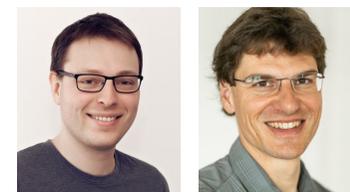
Pushing along x



Roll

# Causal Influence Detection for Reinforcement Learning

(with Maximilian Seitzer and Georg Martius, arXiv:2106.03443)



## Observations

- Real-world agents have limited interventional range
- Causal influence of agent on environment occurs only sparsely

## Idea

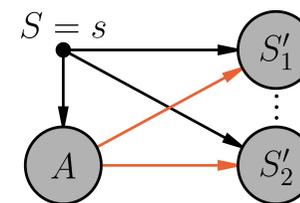
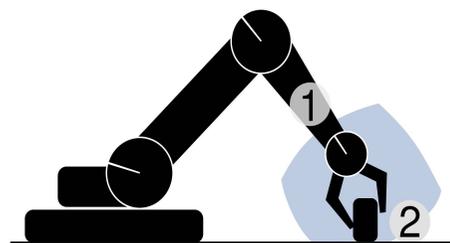
- Use causal influence to speed-up reinforcement learning

## Method

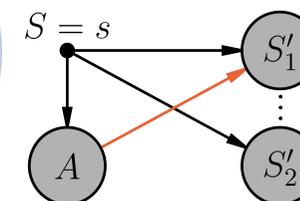
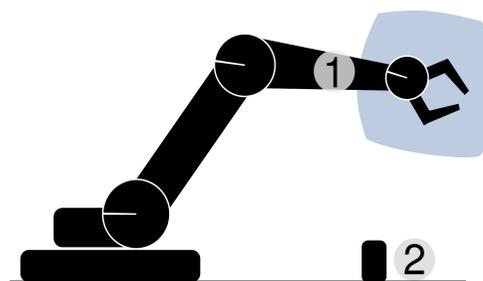
- Define measure of *causal action influence* as a conditional mutual information

$$C(s) := I(S', A | S = s)$$

- Estimate it from data using neural networks



Robot can control object



Causal influence on object impossible

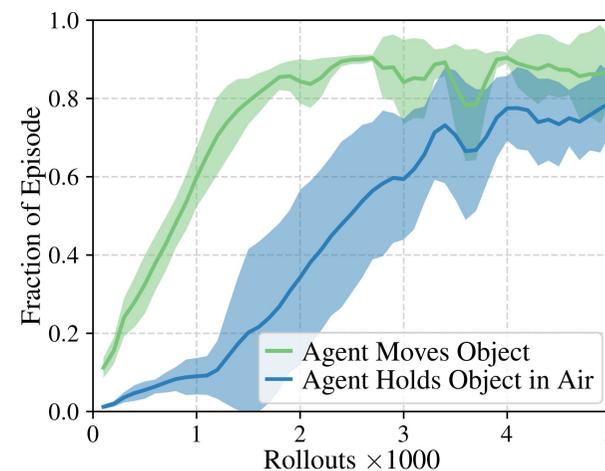
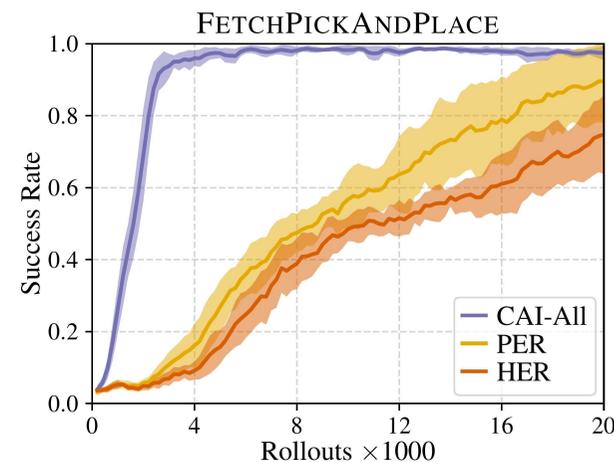
# Causal Influence Detection for Reinforcement Learning

(with Maximilian Seitzer and Georg Martius, arXiv:2106.03443)



## Results

- Focusing on states with causal influence (exploration and prioritization)
  - Highly increased sample-efficiency on robotic manipulation tasks
- Maximizing causal influence as intrinsic motivation
  - Agent quickly discovers interesting behaviors (grasping, lifting)

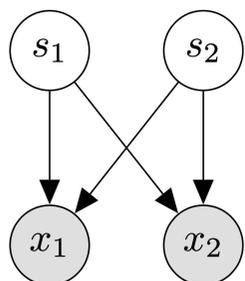


Brockmann et al. OpenAI Gym, arXiv:1606.01540

# Causality for nonlinear ICA

(<https://arxiv.org/abs/2106.05200>)

with Luigi Gresele\*, Julius von Kügelgen\*, Vincent Stimper, Michel Besserve



Observe:

nonlinear mixtures,  $x = f(s)$ , of independent sources  $s$

Goal:

recover the unobserved sources (blind source separation)

Problem:

impossible in general [Hyvärinen & Pajunen, '99]

Recently:

use auxiliary variables [Hyvärinen et al., '16, '17, '19]

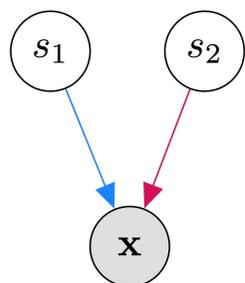
New:

interpret mixing as *causal* process & constrain  $f$  using the ICM principle



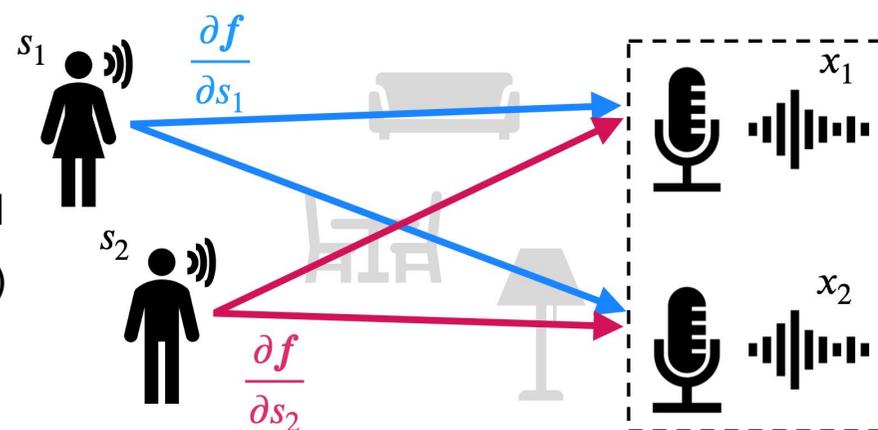
ICM usually applied to **cause distribution**  $p_c$  and **mechanism**  $p_{e|c}$  (or  $f$ ), e.g., cause-effect discovery

But: in nonlinear ICA, cause (source distribution) is unobserved



**Independent mechanism analysis (IMA):**

- ICM at level of mixing function
- contributions  $\frac{\partial f}{\partial s_i}$  of each source to observed distribution be "independent" (not statistical)
- speakers' positions not fine-tuned to room acoustics and microphone placement



# Independent mechanism analysis

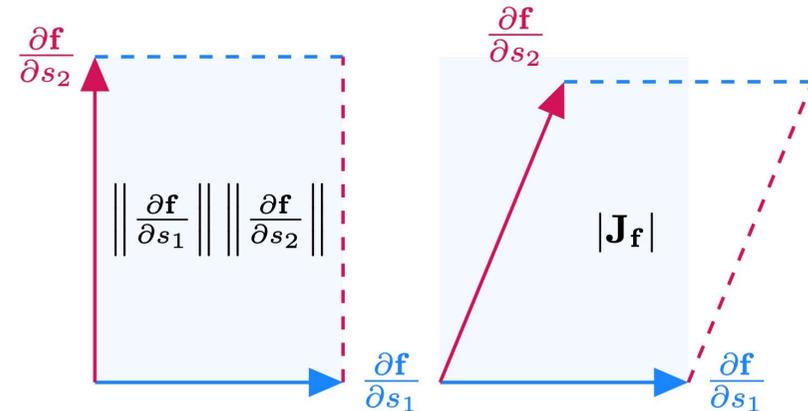
with Luigi Gresele\*, Julius von Kügelgen\*, Vincent Stimper, Michel Besserve



IMA Principle: the influences of each source on the observed distribution are independent in the sense that:

$$\log |\mathbf{J}_f(\mathbf{s})| = \sum_{i=1}^n \log \left\| \frac{\partial \mathbf{f}}{\partial s_i}(\mathbf{s}) \right\|$$

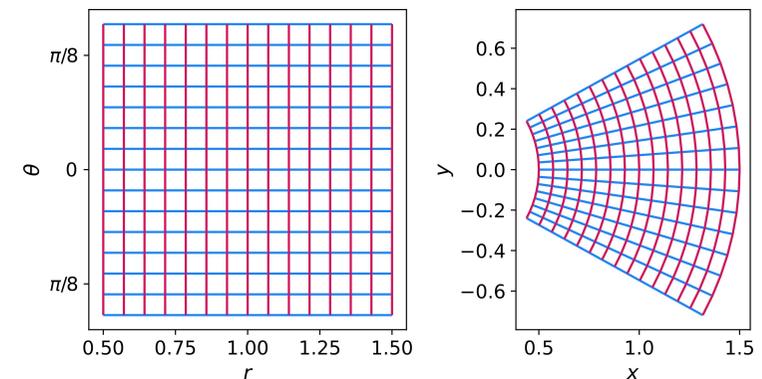
Geometric interpretation: corresponds to an *orthogonality condition* on the columns of the Jacobian.



Contrast function:

$$C_{IMA}(f, p_s) = \int \left( \sum_{i=1}^n \log \left\| \frac{\partial \mathbf{f}}{\partial s_i}(\mathbf{s}) \right\| - \log |\mathbf{J}_f(\mathbf{s})| \right) p_s(\mathbf{s}) d\mathbf{s}$$

- $\geq 0$ , with equality iff.  $f$  is an *orthogonal coordinate transformation*
- *invariant to reparametrisation* of the sources by *permutation* and *element-wise invertible nonlinearities*



# Independent mechanism analysis

with Luigi Gresele\*, Julius von Kügelgen\*, Vincent Stimper, Michel Besserve



## Theory

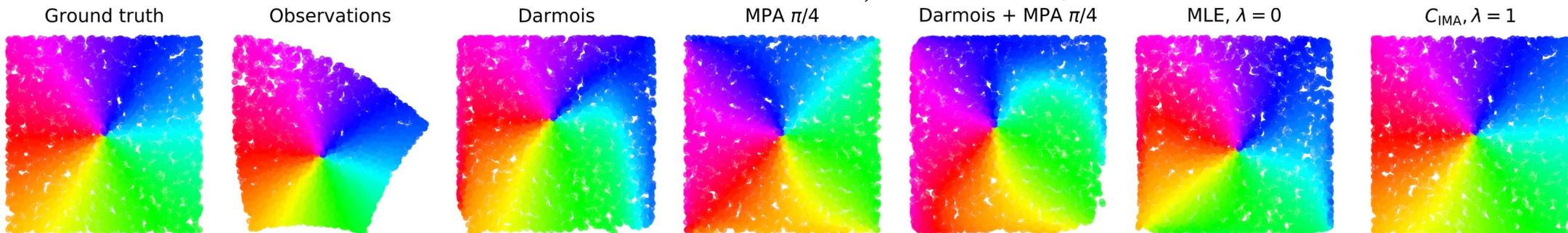
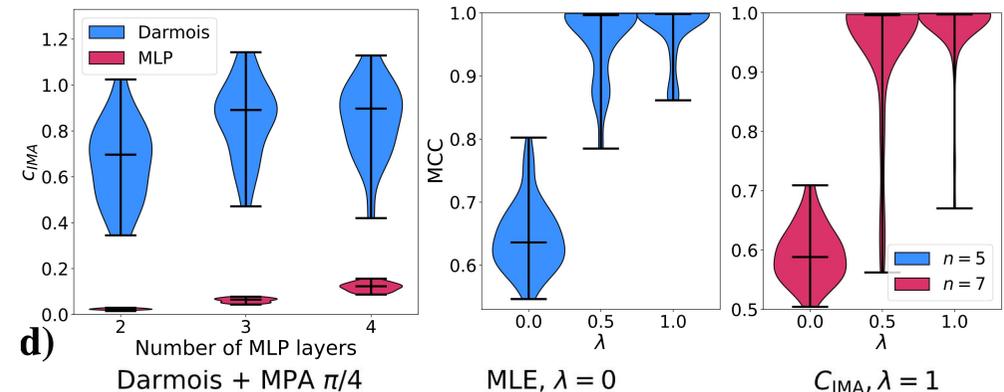
Can rule out (in the sense that  $C_{IMA}$  is larger for) well-known spurious ICA solutions:

- Darmois (inverse CDF) construction
- Measure-preserving automorphisms (MPA)

Consistent with existing identifiability results for linear ICA, and conformal maps.

## Experimental results

Even when assumptions are not perfectly satisfied, IMA seems useful to distinguish spurious solutions and recover the true sources



# Generative models as “causal digital twins”

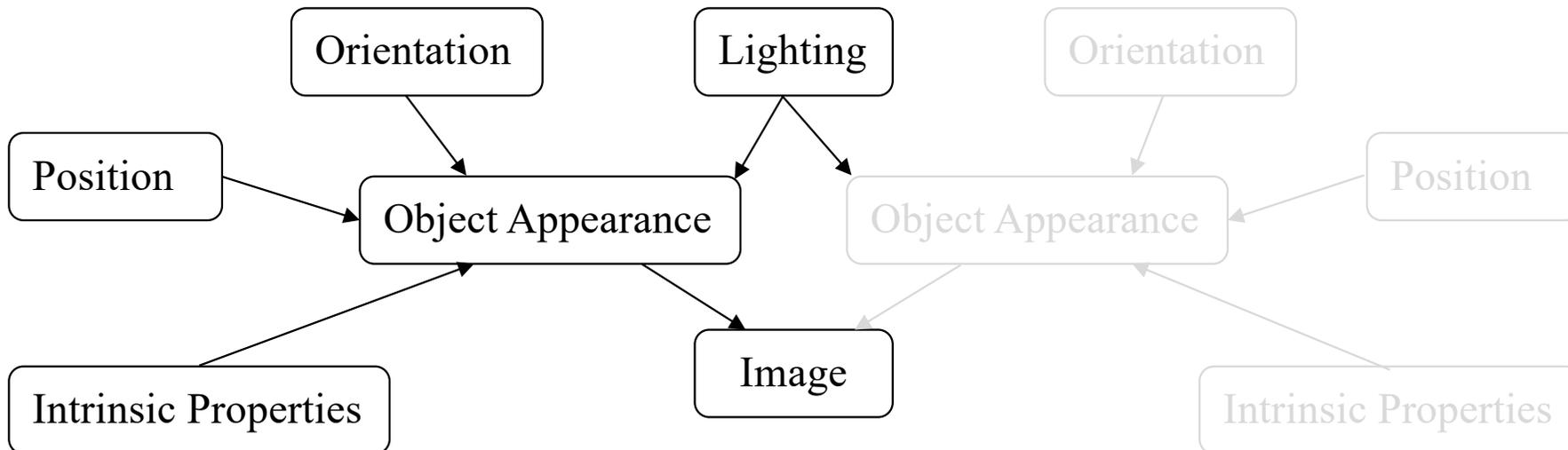
## Disentangled (causal) factorization

<https://arxiv.org/abs/1911.10500>

- independent noises in the causal graph:

$$p(X_1, \dots, X_n) = \prod_{I=1}^n p(X_i | PA_i)$$

- independent mechanisms: changing one  $p(X_i | PA_i)$  does not change the other  $p(X_j | PA_j)$  ( $j \neq i$ ); they remain **invariant** (implies intervenability)





Presented at the ICLR 2020 workshop “Causal learning for decision making”

## TOWARDS CAUSAL GENERATIVE SCENE MODELS VIA COMPETITION OF EXPERTS

**Julius von Kügelgen**<sup>\*†1,2</sup>, **Ivan Ustyuzhaninov**<sup>\*†3</sup>,  
**Peter Gehler**<sup>†4</sup>, **Matthias Bethge**<sup>†3,4</sup>, **Bernhard Schölkopf**<sup>†1,4</sup>

<sup>1</sup>Max Planck Institute for Intelligent Systems Tübingen, Germany

<sup>2</sup>Department of Engineering, University of Cambridge, United Kingdom

<sup>3</sup>University of Tübingen, Germany

<sup>4</sup>Amazon Tübingen, Germany

{jvk,bs}@tuebingen.mpg.de,

{ivan.ustyuzhaninov,matthias.bethge}@bethgelab.org,

pgehler@amazon.com

### ABSTRACT

Learning how to model complex scenes in a modular way with recombining components is a pre-requisite for higher-order reasoning and acting in the physical world. However, current generative models lack the ability to capture the inherently compositional and layered nature of visual scenes. While recent work has made progress towards unsupervised learning of object-based scene representations, most models still maintain a global representation space (i.e., objects are not explicitly separated), and cannot generate scenes with novel object arrangement and depth ordering. Here, we present an alternative approach which uses an inductive bias encouraging modularity by training an ensemble of generative models (*experts*). During training, experts compete for explaining parts of a scene, and thus specialise on different object classes, with objects being identified as parts that re-occur across multiple scenes. Our model allows for controllable sampling of individual objects and recombination of experts in physically plausible ways. In contrast to other methods, depth layering and occlusion are handled correctly, moving this approach closer to a causal generative scene model. Experiments on simple toy data qualitatively demonstrate the conceptual advantages of the proposed approach.

### 1 INTRODUCTION

Proposed in the early days of computer vision (Grenander (1976); Horn (1977)), *analysis-by-synthesis* is an approach to the problem of visual scene understanding. The idea is conceptually elegant and appealing: build a system that is able to synthesize complex scenes (e.g., by rendering), and then understand analysis (inference) as the inverse of this process that decomposes new scenes into their constituent components. The main challenges in this approach are the need for generative models of objects (and their composition into scenes) and the need to perform tractable inference given new

*Tangemann, Schneider et al., 2021*



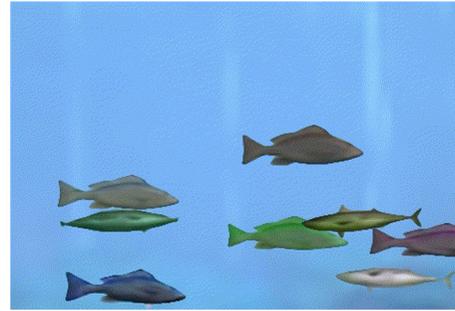
Training set



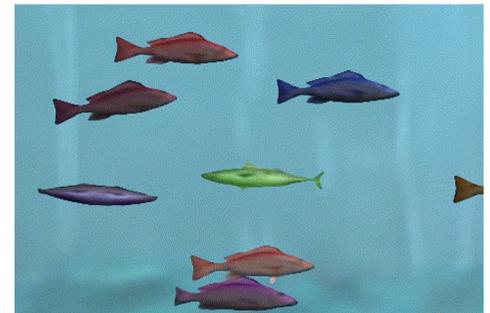
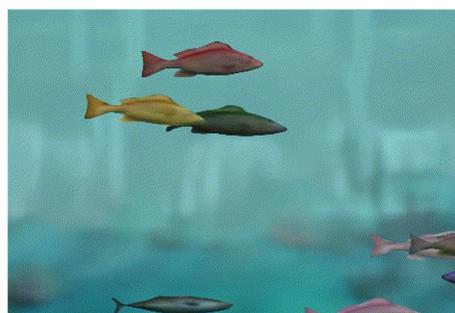
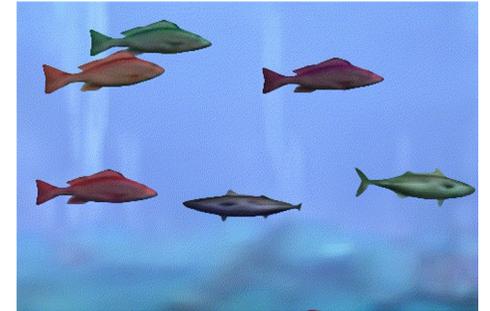
# of objects



fish identities



position



# Towards causal machine learning

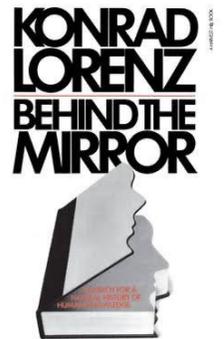
learn *world models* (aka *digital twins*) that are

## (1) data-efficient

- use data from multiple tasks in multiple environments
- use re-usable components that are robust across tasks, i.e., causal (independent) mechanisms
  - disentanglement as a causal problem
  - bias RL to search for invariance / find models where shifts are sparse

## (2) interventional

- move representation learning towards interventional representations: *"thinking is acting is an imagined space"* (Konrad Lorenz) --- planning, reasoning, ...





# Toward Causal Representation Learning

*This article reviews fundamental concepts of causal inference and relates them to crucial open problems of machine learning, including transfer learning and generalization, thereby assaying how causality can contribute to modern machine learning research.*

By BERNHARD SCHÖLKOPF<sup>1</sup>, FRANCESCO LOCATELLO<sup>2</sup>, STEFAN BAUER<sup>1</sup>, NAN ROSEMARY KE, NAL KALCHBRENNER, ANIRUDH GOYAL, AND YOSHUA BENGIO<sup>3</sup>

**ABSTRACT** | The two fields of machine learning and graphical causality arose and are developed separately. However, there is, now, cross-pollination and increasing interest in both fields to benefit from the advances of the other. In this article, we review fundamental concepts of causal inference and relate them to crucial open problems of machine learning, including transfer and generalization, thereby assaying how causality can contribute to modern machine learning research. This also applies in the opposite direction: we note that most work in causality starts from the premise that the causal variables are given. A central problem for AI and causality is, thus, causal representation learning, that is, the discovery of high-level causal variables from low-level observations. Finally, we delineate some implications of causality for machine learning and propose key research areas at the intersection of both communities.

**KEYWORDS** | Artificial intelligence; causality; deep learning; representation learning.

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Bernhard Schölkopf and Stefan Bauer are with the Max Planck Institute for Intelligent Systems, 72076 Tübingen, Germany (e-mail: bs@tuebingen.mpi.de).

## I. INTRODUCTION

If we compare what machine learning can do to what animals accomplish, we observe that the former is rather limited at some crucial feats where natural intelligence excels. These include transfer to new problems and any form of generalization that is not from one data point to the next (sampled from the same distribution), but rather from one problem to the next—both have been termed *generalization*, but the latter is a much harder form thereof, sometimes referred to as *horizontal*, *strong*, or *out-of-distribution* generalization. This shortcoming is not too surprising, given that machine learning often disregards information that animals use heavily: interventions in the world, domain shifts, and temporal structure—by and large, we consider these factors a nuisance and try to engineer them away. In accordance with this, the majority of current successes of machine learning boil down to large-scale pattern recognition on suitably collected *independent and identically distributed (i.i.d.)* data.

To illustrate the implications of this choice and its relation to causal models, we start by highlighting key research challenges.

### A. Issue 1—Robustness



# Connecting Europe



> 1,300  
applications



international  
>70 countries



diverse  
>90 nationalities



competitive  
< 5% acceptance



joint supervision  
>100 Fellows and  
Scholars recruiting



e l l i s

- **120 international exchanges**
- **70 institutions and companies**
- **jointly advised by 116 ELLIS Fellows, Scholars and Members**

**Thank You**

